Name: Key Period: ____________

PC6 - Review **Show all work. Simplify answers.**

Establish the identity.

1) $tan^2\,\theta cos^2\,\theta + cot^2\,\theta sin^2\,\theta = 1$

$\tan^2\theta\cos^2\theta + \cot^2\sin^2\theta$

$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\cancel{\cos^2\theta} + \frac{\cos^2\theta}{\cancel{\sin^2\theta}}\cdot\cancel{\sin^2\theta}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Establish the identity.

2) $\frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta} = \frac{\cos\theta+1}{\sin\theta+1}$

$\frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta}$

$= \left(\frac{1+\sin\theta}{1+\sin\theta}\right)\frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)$

$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} + \frac{\cos\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos^2\theta + \cos\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{\cancel{\cos\theta}(\cos\theta+1)}{\cancel{\cos\theta}(1+\sin\theta)}$

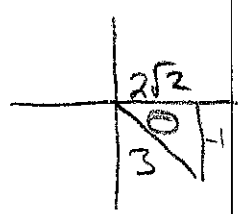
$= \frac{\cos\theta + 1}{\sin\theta + 1}$

Find each of the following

3) $sin^{-1}\left(\frac{1}{2}\right)$

$= \pi/6$

4) $\cot\left(sin^{-1}\left(-\frac{1}{3}\right)\right)$



$\cot(\theta)$

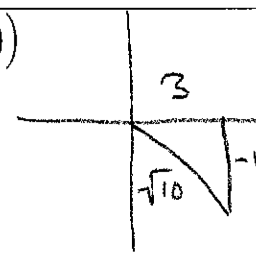
$= \frac{2\sqrt{2}}{-1}$

$\sqrt{8} = \sqrt{4}\cdot\sqrt{2}$

$= 2\sqrt{2}$

$= -2\sqrt{2}$

5) $csc\left(tan^{-1}\left(-\frac{1}{3}\right)\right)$



$\csc(\theta)$

$= \frac{\sqrt{10}}{-1}$

$= -\sqrt{10}$

Solve each equation for θ on the interval $[0, 2\pi]$.

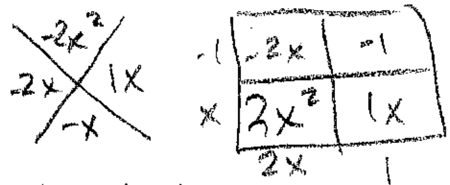
6) $\sqrt{\tan^2\theta} = \sqrt{\frac{1}{3}}$

$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\theta = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

7) $2\sin^2\theta - \sin\theta - 1 = 0$

$2x^2 - x - 1 = 0$



$(2x+1)(x-1) = 0$

$x = -1/2 \quad x = 1 \qquad \sin\theta = -1/2 \quad \sin\theta = 1$

$\theta = \left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

8) $sec(3\theta) = 2$

$\cos(3\theta) = \frac{1}{2} \qquad x = 3\theta$

$\frac{1}{3}x = \theta$

$\cos(x) = \frac{1}{2}$

$x = \left\{ \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \ldots \right.$

$\frac{1}{3}x = \theta$

$\theta = \left\{ \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9} \right\}$

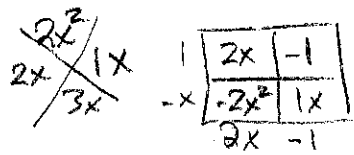
9) $2\sin^2\theta + 3\cos\theta - 3 = 0$

$2(1 - \cos^2\theta) + 3\cos\theta - 3 = 0$

$2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$

$-2\cos^2\theta + 3\cos\theta - 1 = 0$

$-2x^2 + 3x - 1 = 0$



$(2x-1)(-x+1) = 0$

$x = 1/2 \quad x = 1$

$\cos\theta = 1/2 \quad \cos\theta = 1$

$\theta = \left\{ 0, \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

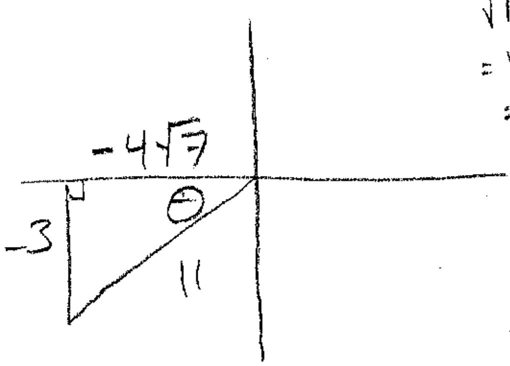
10) Draw a reference triangle given the following:

$\sin\theta = -\frac{3}{11} \quad \tan\theta > 0$

$\sqrt{112}$

$= \sqrt{16} \cdot \sqrt{7}$

$= 4\sqrt{7}$



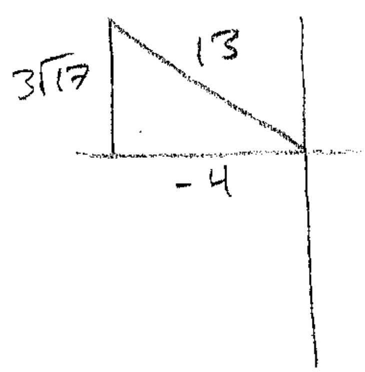
11) Draw a reference triangle given the following:

$cos^{-1}\left(-\frac{4}{13}\right)$

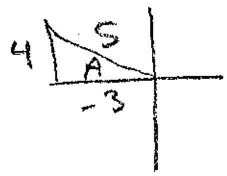
$\sqrt{153}$

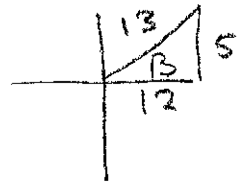
$= \sqrt{9} \cdot \sqrt{17}$

$= 3\sqrt{17}$



12) **Find the exact value of the expression** $tan\left(cos^{-1}\left(-\frac{3}{5}\right)+sin^{-1}\left(\frac{5}{13}\right)\right)$. $= \tan(A+B)$

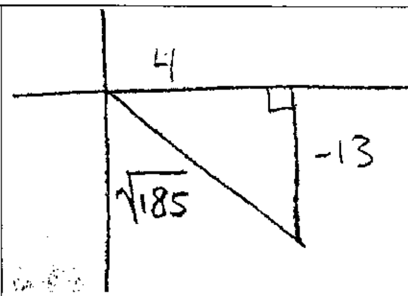




$$\frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\frac{4}{-3}+\frac{5}{12}}{1-\left(\frac{4}{-3}\right)\left(\frac{5}{12}\right)}$$

$$= \frac{\frac{-16}{12}+\frac{5}{12}}{\frac{36}{36}+\frac{20}{36}} = \frac{\frac{-11}{12}}{\frac{56}{36}} = \frac{-11}{12}\cdot\frac{36}{56} = \frac{-396}{672} = \frac{-33}{56}$$

Use the reference triangle to the right on problems 13 and 14.



13) $cos(2\theta)$

$$= \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{4}{\sqrt{185}}\right)^2 - \left(\frac{-13}{\sqrt{185}}\right)^2$$

$$= \frac{16}{185} - \frac{169}{185}$$

$$= \frac{-153}{185}$$

14) $cot(2\theta)$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{-13}{4}\right)}{1-\left(\frac{-13}{4}\right)^2} = \frac{\frac{-26}{4}}{\frac{16}{16}-\frac{169}{16}}$$

$$= \frac{\frac{-26}{4}}{\frac{-153}{16}} = \frac{-26}{4}\cdot\frac{16}{-153}$$

$$= \frac{-416}{-612} = \frac{104}{153}$$

So...

$$\cot(2\theta) = \frac{153}{104}$$

15) Use a half-angle formula to find the exact value of the expression.

$$\sin\left(\frac{7\pi}{8}\right)$$

Quad II

$(+)\sqrt{\frac{1-\cos A}{2}}$

$$\sin\left(\frac{1}{2}\cdot\frac{7\pi}{4}\right) = \sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}}$$

$$= \sqrt{\frac{1-\sqrt{2}/2}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

16) Use a half-angle formula to find the exact value of the expression.

$$\cot\left(\frac{17\pi}{8}\right)$$

$$\tan\left(\frac{1}{2}\cdot\frac{17\pi}{4}\right) = \frac{1-\cos\frac{17\pi}{4}}{\sin\frac{17\pi}{4}}$$

$$= \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{2-\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}$$

So,

$$\cot\left(\frac{17\pi}{8}\right) = \frac{\sqrt{2}}{2-\sqrt{2}}$$

17) Find the exact value of the expression using Sum and Difference formulas.

$$\sec\left(\frac{13\pi}{12}\right)$$

$$\cos\left(\frac{10\pi}{12}+\frac{3\pi}{12}\right) = \cos\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{5\pi}{6}\right)\cos\left(\pi/4\right) - \sin\left(\frac{5\pi}{6}\right)\sin\left(\pi/4\right)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

So ...

$$\sec\left(\frac{13\pi}{12}\right) = \frac{4}{-\sqrt{6}-\sqrt{2}}$$

18) Find the exact value of the expression using Sum and Difference formulas.

$$\tan\left(\frac{17\pi}{12}\right)$$

$$\tan\left(\frac{15\pi}{12}+\frac{2\pi}{12}\right) = \tan\left(\frac{5\pi}{4}+\frac{\pi}{6}\right)$$

$$\frac{\tan\frac{5\pi}{4}+\tan\pi/6}{1-\tan\frac{5\pi}{4}\cdot\tan\pi/6} = \frac{1+\frac{\sqrt{3}}{3}}{1-(1)\left(\frac{\sqrt{3}}{3}\right)}$$

$$= \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3}\cdot\frac{3}{3-\sqrt{3}}$$

$$= \frac{3+\sqrt{3}}{3-\sqrt{3}}$$