

## PC6-4 Identities Notes

Two functions  $f$  and  $g$  are said to be **identically equal** if

$$f(x) = g(x)$$

for every value of  $x$  for which both functions are defined. Such an equation is referred to as an **identity**. An equation that is not an identity is called a **conditional equation**.

**\*Warning\*** Be careful not to treat identities as if they were conditional equations. In an identity, you are trying to prove the equality holds. Multiplying or adding a number to each side is **not** allowed.

**Example 1:** Establish the identity

$$\frac{\cot \theta}{\csc \theta} = \cos \theta$$

Establish the identity means you must show that the equals sign is true.

Step 1: Pick the more complicated side and re-write it.

Step 2: Simplify by re-writing and using your notes to get the other side

$$\frac{\cot \theta}{\csc \theta} = \cos \theta$$

$$\frac{\cot \theta}{\csc \theta} = \frac{\cos \theta}{\frac{1}{\sin \theta}}$$

$$= \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1}$$

$$= \cos \theta$$

**Example 2:** Establish the identity

$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$$

$$\begin{aligned} & 1 - \frac{\sin^2 \theta}{1 - \cos \theta} \\ &= 1 - \frac{(1 - \cos^2 \theta)}{1 - \cos \theta} \\ &= 1 - \frac{(1 + \cos \theta)(1 - \cos \theta)}{1 - \cos \theta} \\ &= 1 - (1 + \cos \theta) \\ &= 1 - 1 - \cos \theta \\ &= -\cos \theta \end{aligned}$$