

PC-1<sup>st</sup> Semester Final Review (Extra Practice Problems)

1) Algebraically find the domain of the following functions.

a)  $h(x) = \frac{3x}{x^2-144}$

$$x^2 - 144 = 0$$

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12$$

$$x = -12$$

$$D: \{x \mid x \neq \pm 12\}$$

b)  $f(x) = \frac{4}{x^2-19x}$

$$x^2 - 19x = 0$$

$$x(x-19) = 0$$

$$x = 0 \quad x = 19$$

$$D: \{x \mid x \neq 0, x \neq 19\}$$

c)  $g(x) = \sqrt{21x+7}$

$$21x + 7 \geq 0$$

$$21x \geq -7$$

$$x \geq \frac{-7}{21} \quad x \geq -\frac{1}{3}$$

$$D: \{x \mid x \geq -\frac{1}{3}\}$$

2) Find the domain of the function.

$$m(x) = \log_4(x+50)$$

$$x+50 > 0$$

$$x > -50$$

$$D: \{x \mid x > -50\}$$

3) Find the domain of the function.

$$n(x) = \log(12x-5)$$

$$12x-5 > 0$$

$$12x > 5$$

$$x > \frac{5}{12}$$

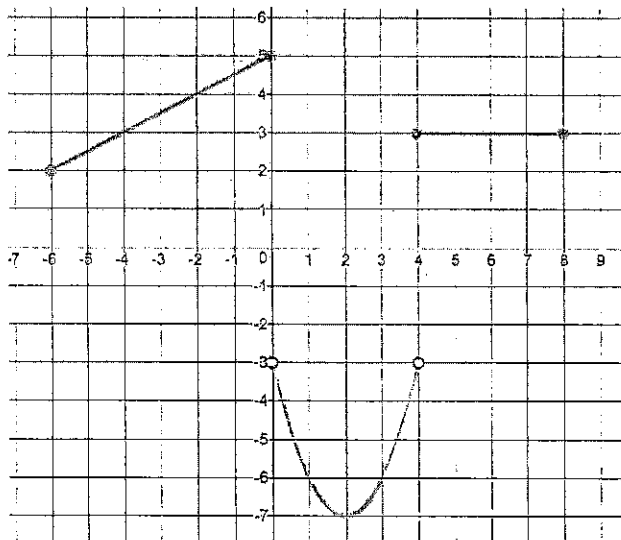
$$D: \{x \mid x > \frac{5}{12}\}$$

4) Find the domain of the function.

$$y = 5^x$$

All real #'s

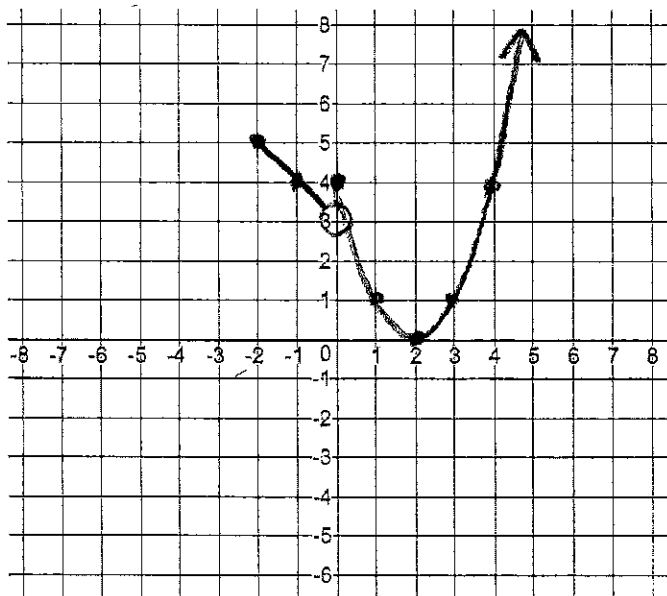
5) Write the definition for the piecewise function.



$$f(x) = \begin{cases} \frac{1}{2}x + 5 & -6 \leq x < 0 \\ (x-2)^2 - 7 & 0 < x < 4 \\ 3 & 4 \leq x \leq 8 \end{cases}$$

6) For the function find the:

$$f(x) = \begin{cases} -x+3 & \text{if } -2 \leq x < 0 \\ 4 & \text{if } x = 0 \\ (x-2)^2 & \text{if } x > 0 \end{cases}$$



7) Fill in the table below. If a value is undefined, write "undefined".

$x$	-6	-1	4	7
$f(x)$	4	0	7	-6
$g(x)$	-1	4	-6	3
$g(x) \cdot f(x)$	-4	0	-42	-18
$f^{-1}(x)$	7	undefined	-6	4
$g^{-1}(x)$	4	-6	-1	undefined

8) Use the graph to the right to answer the following:  
(ESTIMATE where needed)

a) At what value(s) of  $x$ , if any does the graph of  $f$  have a local maximum?

At  $-3.528$  and  $5.738$

b) List the local maximum values.

$19.752$  and  $186.634$

c) At what value(s) of  $x$ , if any does the graph of  $f$  have a local minimum?

At  $0.79$

d) List the local minimum values.

$-361.886$

e) Identify intervals for which the graph is increasing/decreasing.

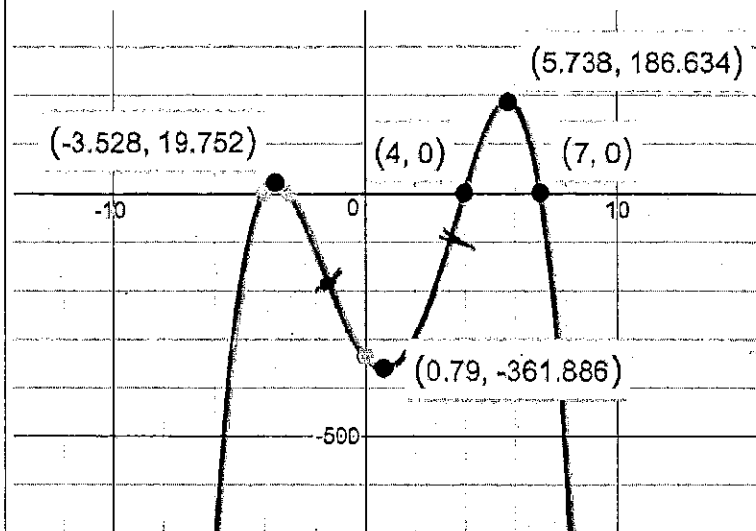
Increasing:  $(-\infty, -3.528) \cup (0.79, 5.738)$

Decreasing:  $(-3.528, 0.79) \cup (5.738, \infty)$

f) Identify intervals for which the graph is concave up/down.

Up:  $(-1.369, 3.264)$

Down:  $(-\infty, -1.369) \cup (3.264, \infty)$



9) Determine if the graph is even, odd, or neither. Explain why/why not.

Neither.

It has no reflection symmetry over the  $y$ -axis (not even). It also has no rotational symmetry around the origin (not odd).

10) Evaluate  $f(-1)$

$\approx -190$

11) Solve  $f(x) = 0$

$x \approx \{-4, -3.5, 4, 7\}$

14) Point  $A(-9, 25)$  is on the graph of  $y = f(x)$ . Determine the location of point  $A'$  after the transformation shown. Describe the transformation steps in detail (use words).

a)  $\frac{1}{5}f(-3(x-7))$

$(-9, 25) \rightarrow$  Divide  $x$  by  $-3$   $(3, 25)$

$(3, 25) \rightarrow$  Add  $7$  to  $x$   $(10, 25)$

$(10, 25) \rightarrow$  Multiply  $y$  by  $\frac{1}{5}$   $(10, 5)$

$(10, 5)$

b)  $5f\left(\frac{1}{3}(x)\right) - 8$

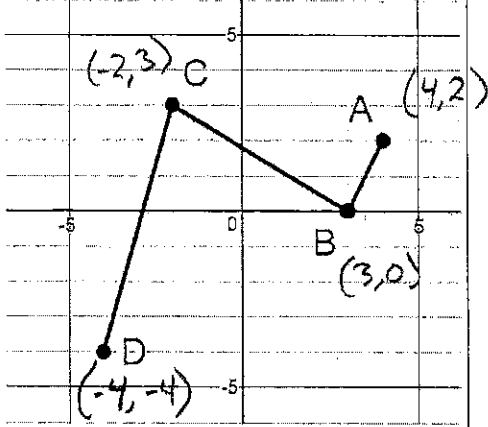
$(-9, 25) \rightarrow$  Multiply  $x$  by  $3$   $(-27, 25)$

$(-27, 25) \rightarrow$  Multiply  $y$  by  $5$   $(-27, 125)$

$(-27, 125) \rightarrow$  Subtract  $8$  from  $y$   $(-27, 117)$

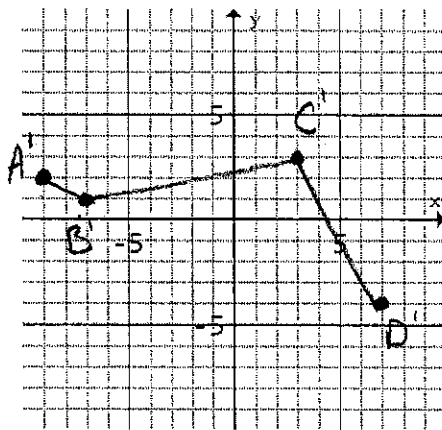
$(-27, 117)$

$h(x)$  is below (#4, #5: 5 pts each)



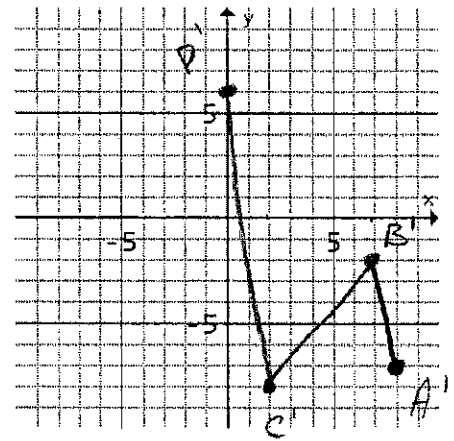
$(x, y) \rightarrow (-2x-1, y)$

15) Graph  $h(-\frac{1}{2}(x+1))$



$(x, y) \rightarrow (x+4, -2y-2)$

16) Graph  $-2h(x-4) - 2$



17) Let  $n(x) = \frac{4x-7}{8x+2}$  Algebraically determine  $n^{-1}(x)$ .

$$x = \frac{4y-7}{8y+2}$$

$$(8y+2)x = 4y-7$$

$$8xy + 2x = 4y - 7$$

$$8xy - 4y = -7 - 2x$$

$$y \frac{8x-4}{8x-4} = \frac{-7-2x}{8x-4}$$

$$n^{-1}(x) = \frac{-7-2x}{8x-4}$$

18) Consider the polynomial function  $f$  with a root of  $3i$ . Find and list all roots.

$$f(x) = x^4 + 4x^3 + 13x^2 + 36x + 36$$

$$(x-3i)(x+3i) = x^2 + 9$$

$-3i$	$3ix$	$9$
$x$	$x^2$	$3ix$
	$x$	$3i$

$9$	$9x^2$	$36x$	$36$
$x^2$	$x^4$	$4x^3$	$4x^2$
	$x^2$	$4x$	$4$

$$(x^2+9)(x^2+4x+4)$$

$4x^2$	$2x$	$4$
$2x$	$x^2$	$2x$
$4x$	$x$	$2$

$$(x^2+9)(x+2)(x+2)$$

Roots

$$x = \{3i, -3i, -2\}$$

19) Given a polynomial with the roots shown below, write a possible polynomial in standard form.

$$x = \{3, 5 + 2i, 5 - 2i\}$$

-2i	2ix	10i	4
5	-5x	25	-10i
X	X <sup>2</sup>	-5x	2ix
	X	-5	2i

$$(X-3)(X^2-10X+29)$$

-3	-3x <sup>2</sup>	30x	-87
X	X <sup>3</sup>	-10x <sup>2</sup>	29x
	X <sup>2</sup>	-10x	29

$$f(x) = X^3 - 13x^2 + 59x - 87$$

20) Given  $R(x) = \frac{3(x-10)(x+2)}{x^2-2x-24}$ , find the following. If none, write 'none'.

a) x-intercept(s)

$$0 = 3(x-10)(x+2)$$

$$\downarrow \quad \downarrow$$

$$x=10 \quad x=-2$$

$$(10, 0) \quad (-2, 0)$$

b) y-intercept(s)

$$\frac{3(0-10)(0+2)}{0^2-2(0)-24}$$

$$= \frac{3(-10)(2)}{-24} = \frac{-60}{-24} = \frac{5}{2}$$

$$(0, \frac{5}{2})$$

c) vertical asymptote(s)

$$x^2-2x-24=0$$

$$(x-6)(x+4)=0$$

$$x=6 \quad x=-4$$

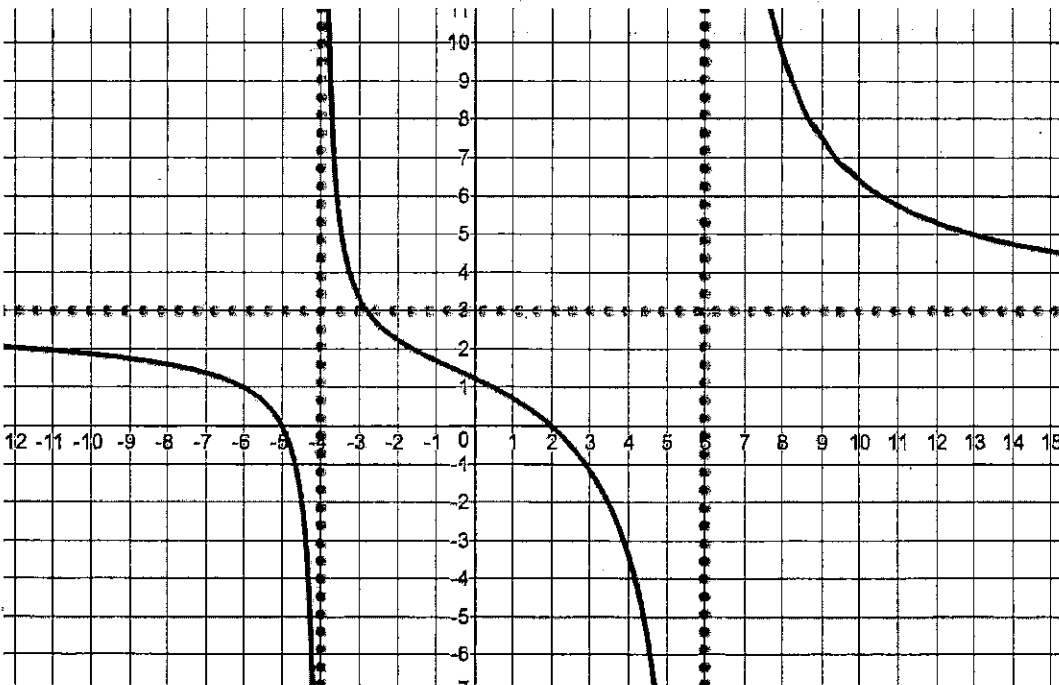
d) Horizontal asymptote

$$y_0=3$$

e) Oblique Asymptote

None

21) Write a possible equation for the graph below. (5 pts)



$$R(x) = \frac{3(x+5)(x-2)}{(x+4)(x-6)}$$

22) Solve for x.

$$5^{x+9} = 625^{4x}$$

$$5^{x+9} = (5^4)^{4x}$$

$$5^{x+9} = 5^{16x}$$

$$x+9 = 16x$$

$$\frac{9}{15} = \frac{15x}{15}$$

$$\frac{3}{5} = x$$

23) Solve for x.

$$\log(2x) + \log(x+1) = \log(12)$$

$$\log(2x^2 + 2x) = \log(12)$$

$$2x^2 + 2x - 12 = 0$$

$$\begin{array}{r|l} \cancel{6x} & \cancel{-24x^2} \\ \cancel{2x} & \cancel{-4x} \end{array} \quad \begin{array}{r|l} -2 & -4x \quad -12 \\ \times & 2x^2 \quad 6x \\ \hline & 2x \quad 6 \end{array}$$

$$(2x+6)(x-2) = 0$$

$$x = -3 \quad x = 2$$

extraneous

24) Solve for x.

$$\log_7(x) + \log_7(3x-14) = 2$$

$$\log_7(3x^2 - 14x) = 2$$

$$7^2 = 3x^2 - 14x$$

$$0 = 3x^2 - 14x - 49$$

$$\begin{array}{r|l} \cancel{21x} & \cancel{-147x^2} \\ \cancel{-14x} & \cancel{-7x} \end{array} \quad \begin{array}{r|l} -7 & -21x \quad -49 \\ \times & 3x^2 \quad 7x \\ \hline & 3x \quad 7 \end{array}$$

$$(3x+7)(x-7) = 0$$

$$x = -\frac{7}{3} \quad x = 7$$

extraneous

25) Alex deposits \$14,322 into an account with a 2.6% interest rate compounded monthly. When will he have \$22,000? Solve algebraically.

$$\frac{22000}{14322} = \frac{14322 \left(1 + \frac{0.026}{12}\right)^{12t}}{14322}$$

$$1.5361 = (1.002167)^{12t}$$

$$\log_{1.002167}(1.5361) = 12t$$

$$\frac{\log(1.5361)}{\log(1.002167)} = 12t$$

$$\frac{198.298}{12} = \frac{12t}{12}$$

$$16.5248 = t$$

In about 16.5248 years

26) An element, Jamiesonian - 33, has a half-life of 3220 years. If there are 555 grams of this element to start, how long until only 230 grams remain?

$$\frac{0.5A_0}{A_0} = \frac{A_0 e^{K(3220)}}{A_0}$$

$$0.5 = e^{K(3220)}$$

$$\ln(0.5) = \frac{K(3220)}{3220}$$

$$K = -0.0002153$$

$$\frac{230}{555} = \frac{555 e^{-0.0002153t}}{555}$$

$$0.4144 = e^{-0.0002153t}$$

$$\frac{\ln(0.4144)}{-0.0002153} = \frac{-0.0002153t}{-0.0002153}$$

$$4091.6098 = t$$

In 4091.6098 years

