

PC 4-3 Notes/Examples

Let $f(x) = \frac{2x+1}{x-1}$. Find the inverse.

$$X = \frac{2y+1}{y-1}$$

$$(y-1)x = 2y+1$$

$$yx - 1x = 2y + 1$$

$$\begin{array}{r} -2y \quad | \quad -2y \\ \hline \end{array}$$

$$yx - 2y - 1x = 1$$

$$\begin{array}{r} +1x \quad | \quad +1x \\ \hline \end{array}$$

$$yx - 2y = 1 + 1x$$

$$y \frac{(x-2)}{x-2} = \frac{1+1x}{x-2}$$

$$f^{-1}(x) = \frac{1+x}{x-2}$$

$$y = \frac{1+x}{x-2} \rightarrow$$

Verify the inverse of $f(x) = \frac{2x+1}{x-1}$ by showing that $f(f^{-1}(x)) = x$

$$f(f^{-1}(x)) = \frac{2(f^{-1}(x)) + 1}{(f^{-1}(x)) - 1}$$

$$= \frac{2\left(\frac{1+x}{x-2}\right) + 1}{\frac{1+x}{x-2} - 1} = \frac{\frac{2+2x}{x-2} + \frac{x-2}{x-2}}{\frac{1+x}{x-2} - \frac{x-2}{x-2}}$$

$$= \frac{2+2x+x-2}{x-2} = \frac{3x}{x-2}$$

$$= \frac{1+x-x+2}{x-2} = \frac{3}{x-2}$$

$$= \frac{3x}{x-2} \cdot \frac{x-2}{3} = \frac{3x}{3} = x$$

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