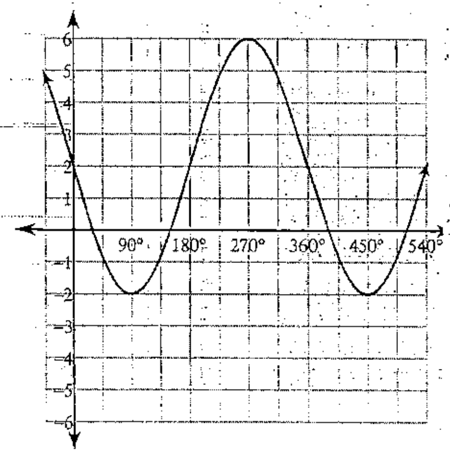
PC 4-3 Inverse Functions

1) Determine if $y = f(x)$ is a well-defined function. Why/Why not?

x	0	1	2	2	3
$f(x)$	-5	-4	3	7	7

No, $x=2$ has more than 1 output

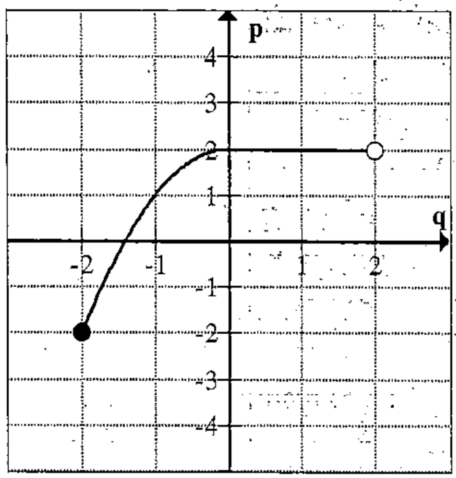
2) Determine if $y = g(x)$ is a well-defined function:

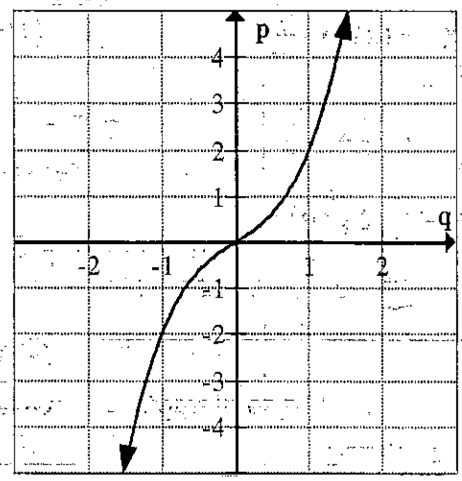


Yes, it passes the vertical line test

A function, $p = f(q)$ is called **one-to-one** if every value of p in the range has a unique value of q in the domain associated to it. In other words, f is one-to-one if $f(x_1) = f(x_2)$ *automatically* implies that $x_1 = x_2$. One-to-one is often abbreviated 1-1, or written in fancy math lingo "injective". The **horizontal line test** states that if every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

3) Determine if the functions are 1-1.





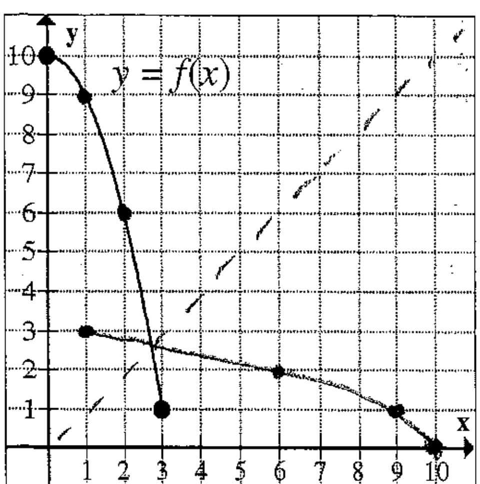
No, fails the horizontal line test

Yes, it passes the horizontal line test

If y is a one-to-one function of x, then x will always be a one-to-one function of y. In this case, if $y = f(x)$, then the function $x = g(y)$ will be a new function where the roles of domain and range have reversed. The function g is called the **inverse function** of f.

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$

4) For the graph of the 1-1 function shown,



(0,10) (1,9) (2,6) (3,1)

(10,0) (9,1) (6,2) (1,3)

a) Evaluate $f(1)$ = 9

b) Evaluate $f^{-1}(1)$ = 3

because $f(3) = 1$

c) Evaluate $f(3)$ = 1

d) Evaluate $f(1)^{-1}$

$= \frac{1}{f(1)} = \frac{1}{9}$

e) Solve $f(x) = 1$

$x = 3$

f) Evaluate $f(3^{-1})$

$= f(\frac{1}{3}) \approx 9.8$

g) Evaluate $f^{-1}(6)$ = 2

h) Sketch $f^{-1}(x)$

5) Let $f(x) = 2x + 1$. Find the inverse.

$x = 2y + 1$

$x - 1 = 2y$

$\frac{x-1}{2} = y$

$f^{-1}(x) = \frac{x-1}{2}$

Domain of f:

All Real #'s

Domain of f^{-1}

All Real #'s

Range of f:

All Real #'s

Range of f^{-1}

All Real #'s

6) Let $g(x) = \frac{x-5}{2x+1}$. Find the inverse.

$x = \frac{y-5}{2y+1}$

$x(2y+1) = y-5$

$2xy + x = y - 5$

$2xy - y = -x - 5$

$y(2x-1) = -x-5$

$y = \frac{-x-5}{2x-1}$ $\quad g^{-1}(x) = \frac{-x-5}{2x-1}$

Domain of g:

$D: \{x \mid x \neq -\frac{1}{2}\}$

Domain of g^{-1}

$D: \{x \mid x \neq \frac{1}{2}\}$

Range of g:

$R: \{y \mid y \neq \frac{1}{2}\}$

Range of g^{-1}

$R: \{y \mid y \neq -\frac{1}{2}\}$

7) The function g is defined in the table shown. Fill in the missing entries.

x	-5	-2	-1	0	2	3	5
$g(x)$	3	-1	5	-5	0	-2	2
$g^{-1}(x)$	0	3	-2	2	5	-5	-1
$g^{-1}(g(x))$	-5	-2	-1	0	2	3	5

Given the graph, graph the inverse function on the same coordinate plane.