PC 4-0 **Definition of Logarithm/Definition of** e

The **logarithm to the base a,** where $a > 0, and\ a \neq 1$ is written as $y = log_a x$.

"y equals the logarithm base a of x"

$y = log_a x$ *if and only if* $x = a^y$

The domain of the logarithm $y = log_a x$ is $x > 0$.

Properties of Logs:

The log of a Product Equals the Sum of the Logs

$$log_a(MN) = log_a M + log_a N$$

Example: $log_3(4x) =$ $\log_3 4 + \log_3 x$

The log of a Quotient Equals the Difference of the Logs

$$log_a\left(\frac{M}{N}\right) = log_a M - log_a N$$

Example: $log_3\left(\frac{4}{x}\right) =$ $\log_3 4 - \log_3 x$

The log of a Power Equals the Product of the Power and the log

$$log_a M^r = r log_a M$$

Example: $log_6 10^8 =$ $8 \cdot \log_6 10$

The **number** e is defined as:

$\left(1 + \frac{1}{n}\right)^n$ as the number n goes to infinity (∞)

In Calculus, using limit notation:

$$e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$$

Fill in the table below to calculate e.

n	$\frac{1}{n}$	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^n$
1	$\frac{1}{1} = 1$	$1 + \frac{1}{1} = 2$	$(2)^1 = 2$
2	$\frac{1}{2}$	$1 + \frac{1}{2} = 1.5$	$(1.5)^2 = 2.25$
5	$\frac{1}{5}$	$1 + \frac{1}{5} = 1.2$	$(1.2)^5 = 2.48832$
10	$\frac{1}{10}$	$1 + \frac{1}{10} = 1.1$	$(1.1)^{10} = 2.59374246$
100	$\frac{1}{100}$	$1 + \frac{1}{100} = 1.01$	$(1.01)^{100} = 2.70481382$
1,000	$\frac{1}{1000}$	$1 + \frac{1}{1000} = 1.001$	$(1.001)^{1000} \approx 2.71692$
10,000	$\frac{1}{10000}$	1.0001	$(1.0001)^{10000} \approx$

2.718