

**PC 2-Review** Show work on all problems

1) Consider the polynomial function  $f$  with a root of  $3i$ . Find and list all roots.

$$f(x) = x^4 + 4x^3 + 13x^2 + 36x + 36$$

$$(x-3i)(x+3i) = x^2+9$$

$3i$	/	9
$x$	$x^2$	/
	$x$	$3i$

9	$9x^2$	$36x$	36
$x^2$	$x^4$	$4x^3$	$4x^2$
	$x^2$	$4x$	4

$$(x^2+9)(x^2+4x+4)$$

$4x^2$	/	4
$2x$	$2x$	/
$x$	$x$	$2$

$$(x^2+9)(x+2)(x+2)$$

Roots:  $x = \{3i, -3i, -2\}$

2) Write the equation of the polynomial in **factored form** given the description:

- a) Degree 7  
Zeros: 4 (multiplicity 2)  
2  
-6 (multiplicity 4)

$$y = (x-4)^2(x-2)(x+6)^4$$

- b) Degree 4  
Zeros:  $7i$   
0  
-12

$$y = (x)(x+12)(x-7i)(x+7i)$$

3) Given a polynomial with the roots shown below, write a possible polynomial in standard form.

$$x = \{3, 5 + 2i, 5 - 2i\}$$

$$(x-3)(x-5-2i)(x-5+2i)$$

$-2i$	/	/	4
$-5$	$-5x$	25	/
$x$	$x^2$	$-5x$	/
	$x$	$-5$	$2i$

$$(x-3)(x^2-10x+29)$$

-3	$-3x^2$	$30x$	-87
$x$	$x^3$	$-10x^2$	$29x$
	$x^2$	$-10x$	29

$$y = x^3 - 13x^2 + 59x - 87$$

4) Given  $g(x) = 3x^4 - 3x^3 + 2x^2 + x + 10$

a) Use Descartes Rule of Signs to determine the number of positive, negative, and imaginary roots.

$$g(x) = 3x^4 - 3x^3 + 2x^2 + x + 10$$

+ - + + +

+	-	i
2	2	0
0	2	2
2	0	2
0	0	4

$$g(-x) = 3x^4 + 3x^3 + 2x^2 - x + 10$$

+ + + - +

b) Use the Rational Zeros Theorem to create a list of potential rational zeros.

$$\frac{10}{3} \Rightarrow \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 3}$$

$\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

5) Given  $h(x) = x^3 - x^2 - 10x - 8$ , determine if  $(x+4)$  is a factor of  $h(x)$ .

$$h(-4) = (-4)^3 - (-4)^2 - 10(-4) - 8 = -48$$

$(x+4)$  is not a factor

6) Write the equation of a polynomial (in factored form) with the following characteristics:

- 1)  $f(-7) = 0 \longrightarrow (x+7)$
- 2)  $(x-10)$  is a factor of the polynomial
- 3)  $(x+3)$  is a factor of the polynomial
- 4)  $f(1) = 5 \longrightarrow (1, 5)$
- 5) The degree of the polynomial is 3.

$$y = a(x+7)(x-10)(x+3)$$

$\begin{matrix} (1, 5) \\ x & y \end{matrix}$

$$5 = a(1+7)(1-10)(1+3)$$

$$5 = a(8)(-9)(4)$$

$$5 = a(-288)$$

$$\frac{-5}{288} = a$$

$$y = \frac{-5}{288}(x+7)(x-10)(x+3)$$

7) Write the equation of the graph shown.

$$y = a(x+7)(x+5)(x-3)^2$$

(0, 6.3)

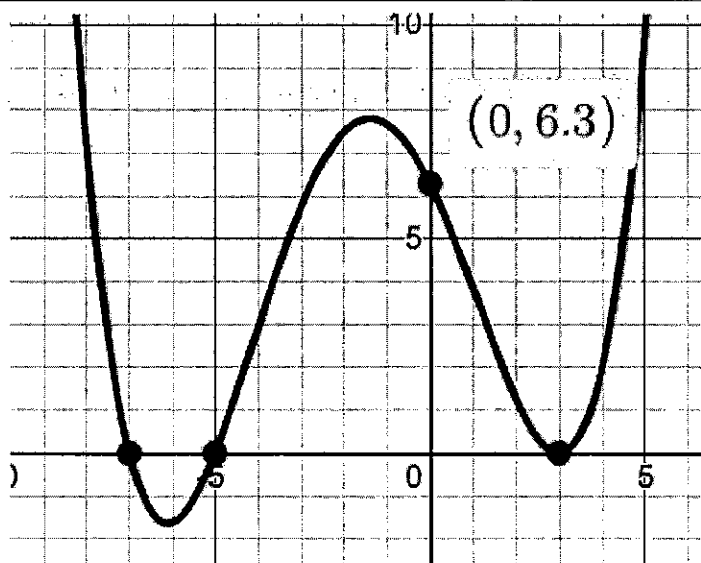
$$6.3 = a(0+7)(0+5)(0-3)^2$$

$$6.3 = a(7)(5)(-3)^2$$

$$6.3 = a(315)$$

$$\frac{1}{50} = a$$

$$y = \frac{1}{50}(x+7)(x+5)(x-3)^2$$



8) Given that  $x=1$  is a root of  $f(x) = x^3 + 2x^2 + 4x - 7$ , find all of the roots.

-1	$-x^2$	$-3x$	$-7$
$x$	$x^3$	$3x^2$	$7x$
	$x^2$	$3x$	$7$

$$(x-1)(x^2 + 3x + 7)$$

$$\begin{matrix} 7x^2 \\ \times \\ 3x \end{matrix} \leftarrow \text{Impossible!}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(7)}}{2(1)} = \frac{-3 \pm \sqrt{-19}}{2}$$

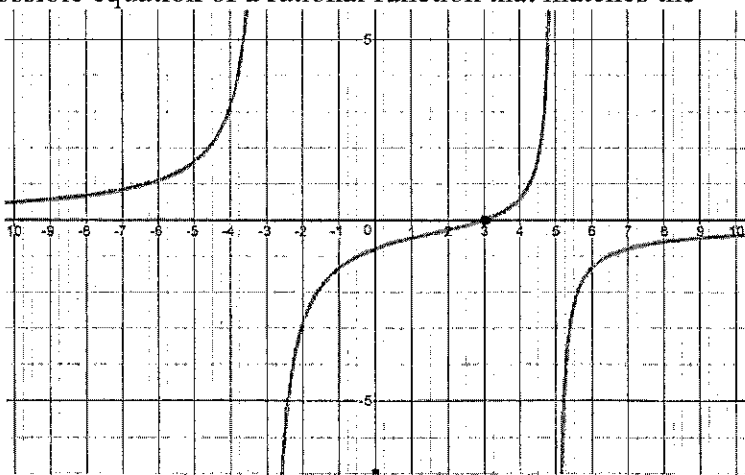
$$x = \frac{-3 \pm i\sqrt{19}}{2}$$

$$\text{Roots: } x = \left\{ 1, \frac{-3 + i\sqrt{19}}{2}, \frac{-3 - i\sqrt{19}}{2} \right\}$$

9) A function has a horizontal asymptote at 6. It also has vertical asymptotes at 4 and -7. It crosses the x-axis at -2. Write the equation that follows those constraints.

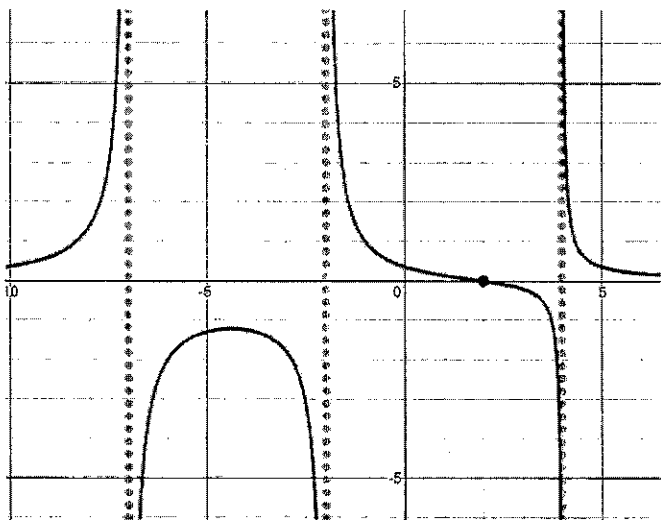
$$y = \frac{6(x+2)^2}{(x-4)(x+7)}$$

10) Write a possible equation of a rational function that matches the graph below.



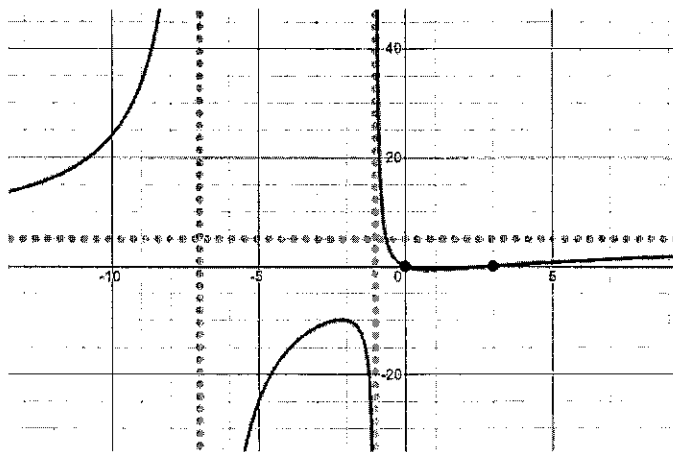
$$y = \frac{x-3}{(x+3)(x-5)}$$

11) Write a possible equation of a rational function that matches the graph below.



$$y = \frac{x-2}{(x+7)(x+2)(x-4)}$$

12) Write a possible equation of a rational function that matches the graph below.



$$y = \frac{5(x)(x-3)}{(x+7)(x+1)}$$

13) Given  $R(x) = \frac{x^2+5x-6}{x^2-x-20}$ , find the following. If none, write 'none'.

a) x-intercept(s)

$$0 = x^2 + 5x - 6$$

$$0 = (x+6)(x-1)$$

$$x = -6 \quad x = 1$$

$$(-6, 0) \quad (1, 0)$$

b) y-intercept.

$$\frac{-6}{-20} = \frac{3}{10}$$

$$(0, \frac{3}{10})$$

c) vertical asymptote(s)

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$x = 5 \quad x = -4$$

d) Horizontal asymptote

$$y = 1$$

e) Oblique Asymptote

None

f) POI with the graph and any horizontal/oblique asymptotes

$$1 = \frac{x^2 + 5x - 6}{x^2 - x - 20}$$

$$(-\frac{7}{3}, 1)$$

$$x^2 - x - 20 = x^2 + 5x - 6$$

$$-x - 20 = 5x - 6$$

$$-20 = 6x - 6$$

$$-14 = 6x$$

$$-\frac{7}{3} = x$$

g) Graph  $R(x)$  using your answers.

