

Students will write the equation of the horizontal or oblique asymptote of a rational function. Students will find the point of intersection between a graph and its horizontal asymptote.

Directions: Write the equation of the horizontal or oblique asymptote.

1st: Look at the degree of the polynomial in the numerator AND the denominator

Example:

| | | | |
|----------|---|------------------|---------|
| Degree 2 | → | $4x^2 + 2x - 5$ | $N = 2$ |
| Degree 4 | → | $3x^4 + 5x - 10$ | $D = 4$ |

2nd: Using the degree, determine which category it fits in

Which Category???

The numerator degree (N) is less than the denominator degree (D).
 $N < D$

Answer: The horizontal asymptote is:

$y = 0$

Example: $R(x) = \frac{4x^2 + 2x - 5}{3x^4 + 5x - 10}$

$2 < 4$
so

Horizontal Asymptote is

$y = 0$

Example:

$f(x) = \frac{x}{x^2 + 1}$

Example:

$g(x) = \frac{x^{17} + x^3 + 5}{x^{18} + x^2}$

The numerator degree (N) is the same as the denominator degree (D).
 $N = D$

Answer: The horizontal asymptote is the fraction formed from the numerator's leading coefficient divided by the denominator's leading coefficient.

$y = \frac{\text{leading coefficient}}{\text{leading coefficient}}$

Example: $R(x) = \frac{4x^5 - 8x^2 - 15}{3x^5 + 5x - 9}$

Horizontal Asymptote: $y = \frac{4}{3}$

Example:

$y = \frac{7x^{10} + 3x^2}{14x^{10} + 5}$ $y = \frac{1}{2}$

Example:

Numerator degree (N) is one more than the denominator degree (D).
 $N = D + 1$

Answer: There is **no** horizontal asymptote!!!! There is an oblique (slanted) asymptote. You must divide to find the equation of the oblique asymptote!

$3 = 2 + 1$

Example: $\frac{4x^3 + 2x - 10}{x^2 + 1}$

Divide $4x^3 + 2x - 10$ by $x^2 + 1$. The division won't work completely and you'll have a remainder. Keep only the quotient portion as your equation:

| | | |
|-------|--------|-------|
| 1 | $4x$ | |
| x^2 | $4x^3$ | $-2x$ |

$4x$

I can't continue.

The oblique asymptote is

$y = 4x$

Find the horizontal or oblique asymptotes for each rational function below

$R(x) = \frac{4x^2 + x - 4}{4x^2 + 5x + 3}$

$y = 1$

$R(x) = \frac{x^3 - 2x + 6}{x^4 - 7x^2 + 7x - 16}$

$y = 0$

$R(x) = \frac{3x^3 - 4x - 2}{2x^3 + 5x}$

$y = \frac{3}{2}$

$R(x) = \frac{3x^3 - 2x - 5}{x^2 + 2}$

| | | |
|-------|--------|-------|
| 2 | $6x$ | |
| x^2 | $3x^3$ | $-8x$ |

$3x$?

$y = 3x$

$R(x) = \frac{2x^2+2x-3}{x^2-4x-4}$ has a horizontal asymptote. It is: $y=2$ Find the point of intersection (if it exists) between the graph and the asymptote. Graphs can, but don't always, cross the horizontal/oblique asymptotes.

$(x^2-4x-4) \cdot 2 = \frac{2x^2+2x-3}{x^2-4x-4} \cdot (x^2-4x-4)$

$2(x^2-4x-4) = 2x^2+2x-3$

$2x^2-8x-8 = 2x^2+2x-3$

$-8x-8 = 2x-3$
 $-8 = 10x-3$
 $-5 = 10x$

$-\frac{1}{2} = x$

P.O.I.
 $(-\frac{1}{2}, 2)$

Given $R(x) = \frac{4x^2+4x-3}{3x^2-4x-4}$, sketch a graph using the information given below. To save time, I've found most of the information for you. ☺

a) x-intercept(s)

$(-\frac{3}{2}, 0)$ $(\frac{1}{2}, 0)$

b) y-intercept

$(0, \frac{3}{4})$

c) vertical asymptote(s)

$x = \frac{2}{3}$ $x = 2$

d) horizontal asymptote(s)

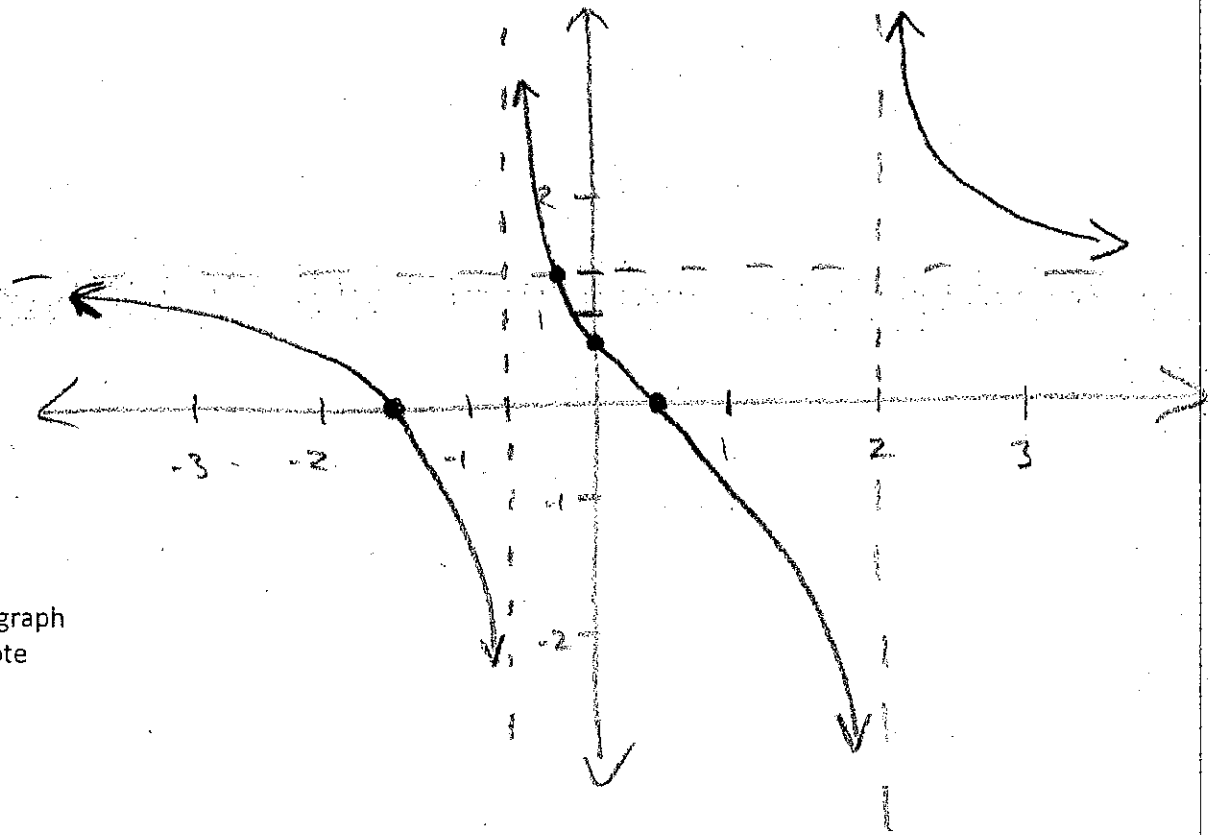
$y = \frac{4}{3}$

e) oblique asymptote(s)

None

f) intersection point of the graph and the horizontal asymptote

$(-\frac{1}{4}, \frac{4}{3})$



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