

Students will identify x and y intercepts, vertical asymptotes, and holes in the graph of a rational function.

A rational function $R(x)$ is a function that can be expressed as a quotient of two polynomials. Rational functions can have x- and y- intercepts like any other function, and they very often have several horizontal, vertical, or oblique (slanted) asymptotes. We will be finding all of these characteristics **algebraically** and using them to construct a careful graph.

Example of Rational Functions:

Non-Examples of Rational Functions:

Find the x-intercepts:

$$f(x) = \frac{3x^2 - 13x - 10}{x^2 - x - 6}$$

Find the y-intercepts:

$$f(x) = \frac{3x^2 - 13x - 10}{x^2 - x - 6}$$

Graph your findings:

For each rational function below, find the x and y intercepts algebraically.

$$R(x) = \frac{x+1}{x^2+x-2}$$

$$R(x) = \frac{x^2-4x+3}{x^2+7x-4}$$

$$R(x) = \frac{x^3-2x}{5x^2-x-4}$$

$$R(x) = \frac{x^2-12x+20}{3x^2-5x-10}$$

$$R(x) = \frac{x^2-12}{x+3}$$

$$R(x) = \frac{5x^2+6x+1}{x+3}$$

The **Domain** of a function is the set of all possible x -values. To find the domain of a rational function, determine when the denominator will be 0.

Example: $f(x) = \frac{43}{(x+4)(x-5)}$

$$(x + 4)(x - 5) = 0$$

$$x + 4 = 0$$

$$-4 \quad -4$$

$$x = -4$$

$$x - 5 = 0$$

$$+5 \quad +5$$

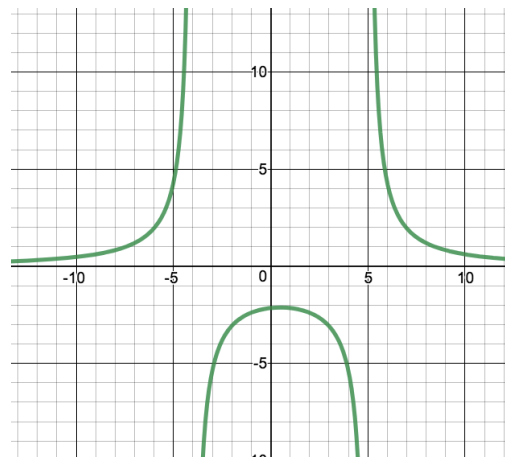
$$x = 5$$

$$D: \{x | x \neq -4, x \neq 5\}$$

Finding the **vertical asymptotes** involves essentially the same process (set the denominator equal to 0).

Investigate the graph of this function to the right.

The graph of the same function $f(x) = \frac{43}{(x+4)(x-5)}$ looks like this:



There are **two vertical asymptotes**. They are:

$$x = -4 \text{ and } x = 5$$

For each rational function below, find the equation of any vertical asymptotes algebraically.

$$R(x) = \frac{x+2}{x^2+4x+3}$$

$$R(x) = \frac{x^3-2x}{x^2-16}$$

$$R(x) = \frac{x^2+x+1}{3x^3+8x^2+4x}$$

Glue Goes Here

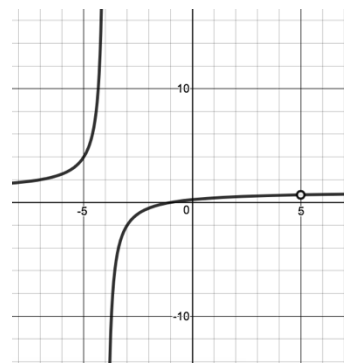
PC 3-5 THE SPECIAL CASE

Students will understand the special case in which a hole in the graph is caused by a repeated factor.

Although finding the vertical asymptotes means investigating the denominator, there are cases where examining the numerator is necessary. If the numerator and the denominator both have the same factor, a hole in the graph (an undefined x -value) occurs.

Example: $g(x) = \frac{x^2-4x-5}{x^2-x-20}$

The graph of $g(x) = \frac{x^2-4x-5}{x^2-x-20}$



In addition to the hole in the graph, the x -intercepts are also changed. Normally, the graph of $g(x)$ would have x -intercepts of $(-1,0)$ and $(5,0)$. Instead, we can see from the graph the x -value of 5 has been eliminated. Proceed with caution when determining characteristics of a rational function especially when a repeated factor occurs.

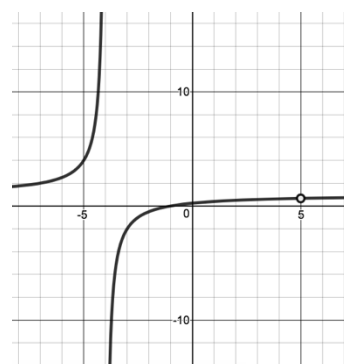
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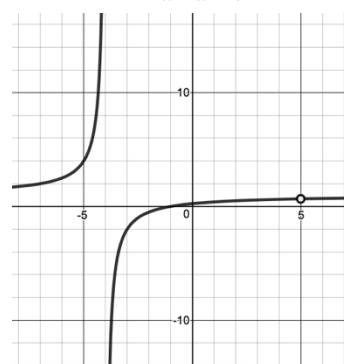
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