PC 3-4 Notes

I can apply Descartes Rule of Signs to determine the number of positive, negative, and imaginary zeros of a polynomial. I can apply the Rational Zeros Theorem to a polynomial function to determine the potential Rational Zeros.

Descartes Rule of Signs:

Let f denote a polynomial function written in standard form.

Given a polynomial with integer coefficients written in standard form, the number of positive real zeros of f either equals the number of variations in the sign of the nonzero coefficients of $f(x)$ or else equals that number less an even integer.

The number of negative real zeros of f either equals the number of variations in the sign of the nonzero coefficients of f(-x) or else equals that number less an even integer.

Example Application of Descartes Rule of Signs:

$$f(x) = x^3 - 4x^2 + 5x - 8$$

+ − + −

$f(-x) = -x^3 - 4x^2 - 5x - 8$

− − − −

+	−	i
3	0	0
1	0	2

Given the polynomial below, use Descartes Rule of Signs to determine the number of positive, negative and imaginary roots.

1) $f(x) = x^3 + 6x^2 + 11x + 6$

+ + + +

$f(-x) = -x^3 + 6x^2 - 11x + 6$

− + − +

+	−	i
0	3	0
0	1	2

2) $f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5$

+ − + + − +

$f(-x) = -x^5 - x^4 - 3x^3 + 9x^2 + x + 5$

− − − + + +

+	−	i
4	1	0
2	1	2
0	1	4

3) $f(x) = 2x^3 - 2x^2 + x + 5$

+ − + +

$f(-x) = -2x^3 - 2x^2 - x + 5$

− − − +

+	−	i
2	1	0
0	1	2

4) $f(x) = 2x^3 + 11x^2 - 7x - 6$

+ + − −

$f(-x) = -2x^3 + 11x^2 + 7x - 6$

− + + −

+	−	i
1	2	0
1	0	2

5) $f(x) = 7x^5 - 3x^3 + x - 20$

+ − + −

$f(-x) = -7x^5 + 3x^3 - x - 20$

− + − −

+	−	i
3	2	0
1	2	2
1	0	4
3	0	2

6) $f(x) = x^5 + 3x^4 + x^3 - x^2 + x - 1$

+ + + − + −

$f(-x) = -x^5 + 3x^4 - x^3 - x^2 - x - 1$

− + − − − −

+	−	i
3	2	0
1	2	2
1	0	4
3	0	2

Rational Zeros Theorem

Let f be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

and $a_n \neq 0, a_0 \neq 0$

where each coefficient is an integer. If $\frac{p}{q}$, in lowest terms is a rational zero of f, then p must be a factor of a_0, and q must be a factor of a_n.

Example application of the Rational Zeros Theorem:

$$f(x) = 1x^4 - 4x^3 - 7x^2 + 22x + 24$$

$q = 1$ $\qquad$ $p = 24$

$$\frac{24}{1} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}{\pm1}$$

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Given the polynomial below, create a list of all the potential rational zeros using the Rational Zeros Theorem.

1) $f(x) = 3x^3 + 4x^2 - 5x - 2$

$\frac{-2}{3} \Rightarrow \frac{\pm1 \quad \pm2}{\pm1 \quad \pm3}$

$\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}$

2) $f(x) = 2x^3 + 3x - 5$

$\frac{-5}{2} \quad \frac{\pm1 \quad \pm5}{\pm1 \quad \pm2}$

$\pm1, \pm\frac{1}{2}, \pm5, \pm\frac{5}{2}$

3) $f(x) = 2x^3 + 11x^2 - 7x - 6$

$\frac{-6}{2} \quad \frac{\pm1, \pm2, \pm3, \pm6}{\pm1 \quad \pm2}$

$\pm1, \pm\frac{1}{2}, \pm2, \pm3, \pm\frac{3}{2} \pm6$

4) $f(x) = 3x^3 - 7x^2 - 14x + 24$

$\frac{24}{3} \quad \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}{\pm1, \pm3}$

$\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm3, \pm4, \pm\frac{4}{3}, \pm6, \pm8, \pm\frac{8}{3}, \pm12, \pm24$

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