Key

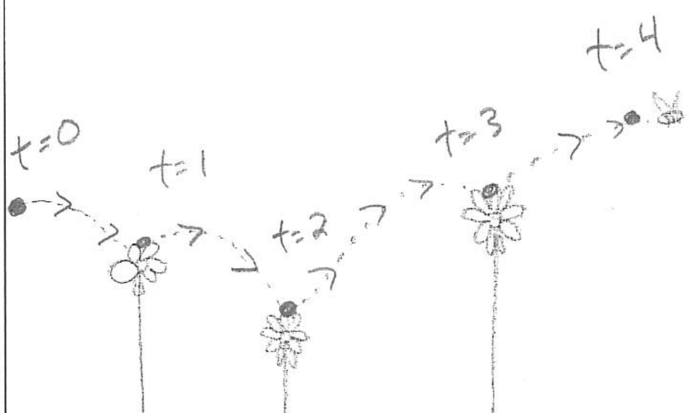
PC 8-3 **Parametric Equations Investigation**

Parametric equations are equations that define both the x-coordinate and y-coordinate independently. Each as a separate function based on a single variable, ***t***. We think of ***t*** as being the time and the x and y coordinates being the path that one follows as time progresses. Remember, both x and y are functions of ***t***.

Graphing parametric equations: For the parametric equations given, complete the table and plot the points. Indicate the time on the graph and draw arrows to indicate the orientation.

As time (***t***) progresses towards more positive values, the new coordinates (x,y) created provide a pathway for the shape of the parametric equation. It is for this reason that when graphing a parametric equation we use an arrow to indicate the direction of the shape.

Example: A honey bee is flying from one flower to the next. The picture shows the path of the bee at different points (x,y) as time (***t***) moves forward.

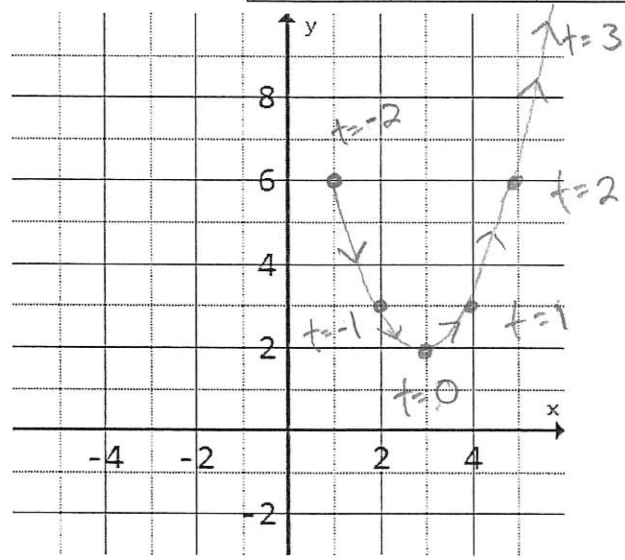


Graph the parametric equations by filling out the table.

$$\begin{cases} x(t) = t + 3 \\ y(t) = t^2 + 2 \end{cases}$$

$$-2 \le t \le 3$$

t	$x(t)$	$y(t)$
-2	1	6
-1	2	3
0	3	2
1	4	3
2	5	6
3	6	11



Circle Equations

The graph of the parametric equations

$$\begin{cases} x(t) = h + r\cos(t) \\ y(t) = k + r\sin(t) \end{cases} \quad \text{for } 0 \le t \le 2\pi$$

is a circle with radius __r__ and center at __(h, k)__. The rectangular equation for such a circle is

$$(x-h)^2 + (y-k)^2 = r^2.$$

Implicit ↗

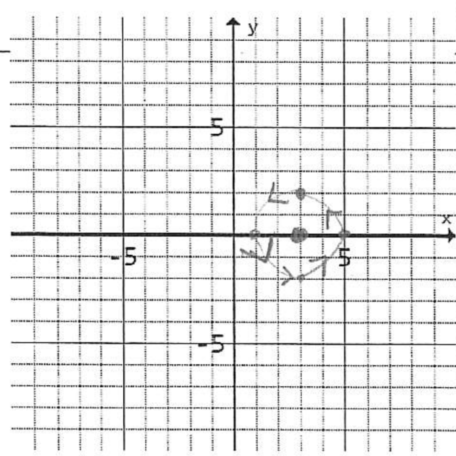
Graph.

$$\begin{cases} x(t) = 3 + 2\cos(t) \\ y(t) = 2\sin(t) \end{cases}$$

Center (3,0)

Write the rectangular Equation below:

$(x-3)^2 + (y-0)^2 = 4$



Ellipse Equations

The graph of the parametric equations

$$\begin{cases} x(t) = h + a\cos(t) \\ y(t) = k + b\sin(t) \end{cases} \quad \text{for } 0 \le t \le 2\pi$$

is an ellipse with center at (**h**,**k**) and ***a*** determines the horizontal "radius" and ***b*** determines the vertical "radius". The rectangular equation for an ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

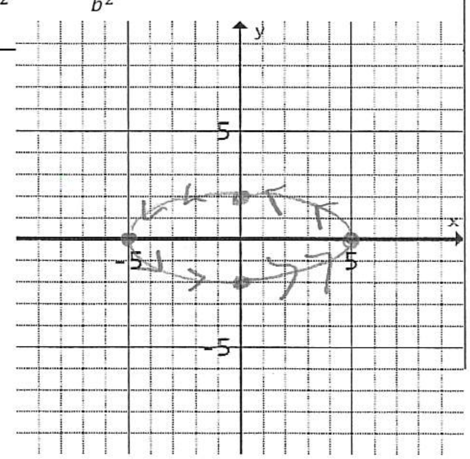
Graph.

$$\begin{cases} x(t) = 5\cos(t) \\ y(t) = 2\sin(t) \end{cases}$$

Center (0,0)

Write the rectangular Equation below:

$$\frac{(x-0)^2}{25} + \frac{(y-0)^2}{4} = 1$$



-- Fold this paper and glue it in your notebook --

Implicit equations:

Rectangular equations like the one for the ellipse and a circle, where *y* is not explicitly solved for, are called implicit equations. In other words, any rectangular equation where x and y are on the same side is considered implicit.

Practice: Remove the parameter *t* in the parametric equations below and re-write as an implicit equation.

Parametric Equation:	**Implicit Equation:**	**Parametric Equation:**	**Implicit Equation:**
$\begin{cases} x(t) = t + 3 \\ y(t) = t^2 \end{cases}$ $X = t+3$ Solve for t. $X - 3 = t$. Plug $t = x-3$ into the y equation $y = (x-3)^2$	$y - (x-3)^2 = 0$	$\begin{cases} x(t) = 2 + 7\cos(t) \\ y(t) = -3 + 4\sin(t) \end{cases}$ $7^2 = 49$ $4^2 = 16$ $(h, K) \to (2, -3)$	$\frac{(X-2)^2}{49} + \frac{(y+3)^2}{16} = 1$