Name: Key Period: ________

PC Second Semester Review

Show your work in each problem below – your work is part of your answer. Always simplify when possible.	**Exact Answer:** $3\sqrt{5}$ (Simplified)	**Approximate Answer:** 6.71 (round)

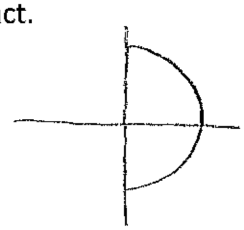
1. Determine the exact value of the function below.
Exact Answer. $sin^{-1}(-\frac{1}{2})$

$= -30^\circ = -\frac{\pi}{6}$

2. Determine the exact value of the function below.
Exact Answer. $cos^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$

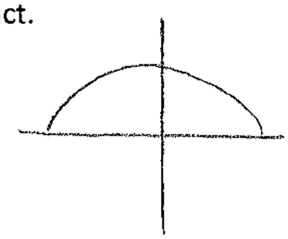
$= \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} = 120^\circ$

3. In which quadrants does $sin^{-1}(x)$ exist? Draw a picture to illustrate this fact.



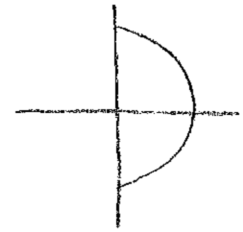
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

4. In which quadrants does $cos^{-1}(x)$ exist? Draw a picture to illustrate this fact.



$0 \le \theta \le \pi$

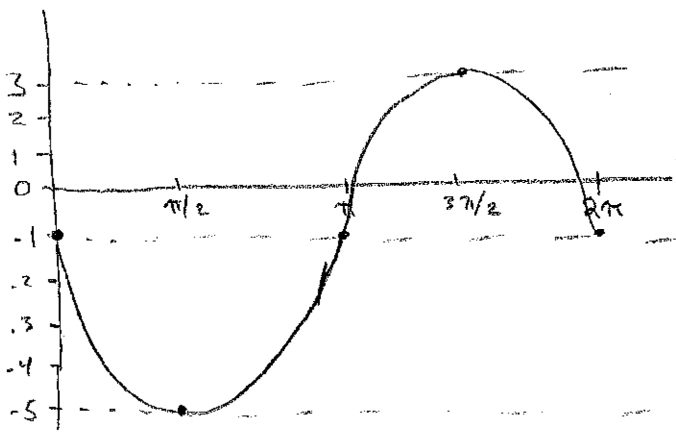
5. In which quadrants does $tan^{-1}(x)$ exist? Draw a picture to illustrate this fact.



$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

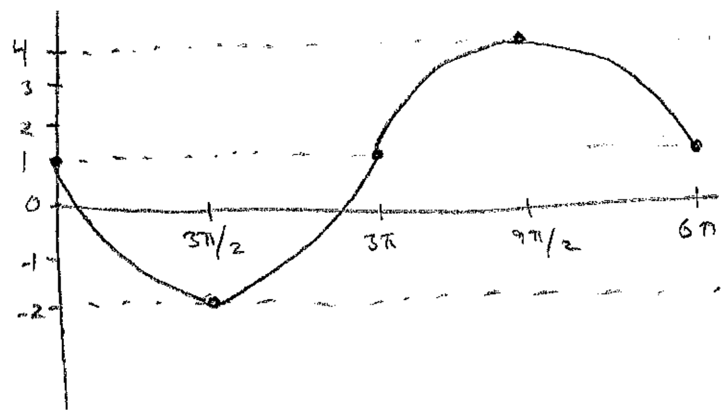
6. Graph one cycle of the function below.

$$y = 4cos\left(x + \frac{\pi}{2}\right) - 1$$



7. Graph one cycle of the function below.

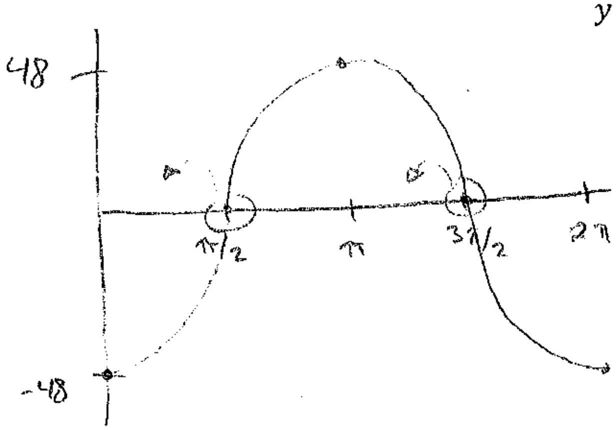
$$y = -3\sin\left(\frac{1}{3}x\right) + 1$$



8. Use a graph to determine the zeros on the interval $[0,2\pi]$. **Exact Answer.**

$$y = -48\cos\left(x + \frac{\pi}{13}\right)$$

Moves every point left π/13. So you should subtract π/13 from the zeros



$\frac{\pi}{2} - \frac{\pi}{13} = \frac{13\pi}{26} - \frac{2\pi}{26} = \frac{11\pi}{26}$

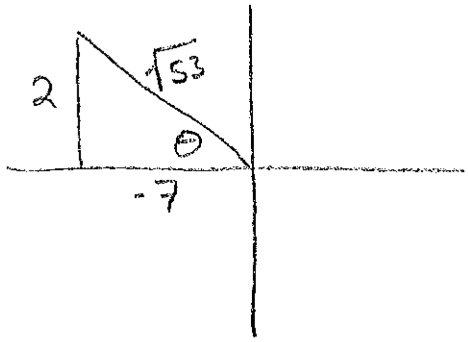
$\frac{3\pi}{2} - \frac{\pi}{13} = \frac{39\pi}{26} - \frac{2\pi}{26} = \frac{37\pi}{26}$

$X = \left\{\frac{11\pi}{26}, \frac{37\pi}{26}\right\}$

9. Given $\tan\theta = -\frac{2}{7}$ and $\sin\theta > 0$. **Exact Answer.**

a) Draw a reference triangle

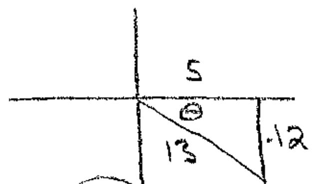
b) Find the value of all six trig functions.



$\sin\theta = \frac{2}{\sqrt{53}} = \frac{2\sqrt{53}}{53}$	$\csc\theta = \frac{\sqrt{53}}{2}$
$\cos\theta = \frac{-7}{\sqrt{53}} = \frac{-7\sqrt{53}}{53}$	$\sec\theta = \frac{\sqrt{53}}{-7}$
$\tan\theta = -\frac{2}{7}$	$\cot\theta = -\frac{7}{2}$

10. Determine the **exact** value of the function below.

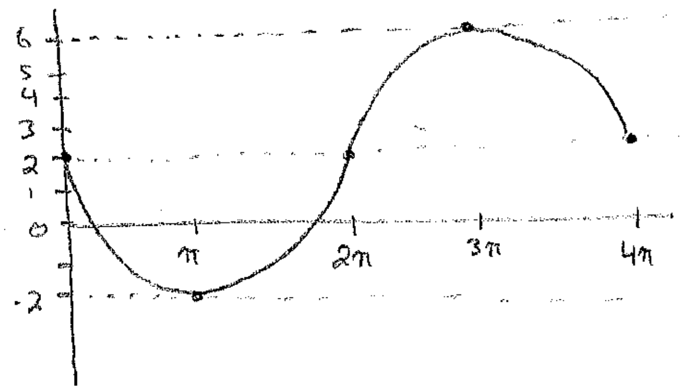
$$csc\left(tan^{-1}\left(-\frac{12}{5}\right)\right)$$



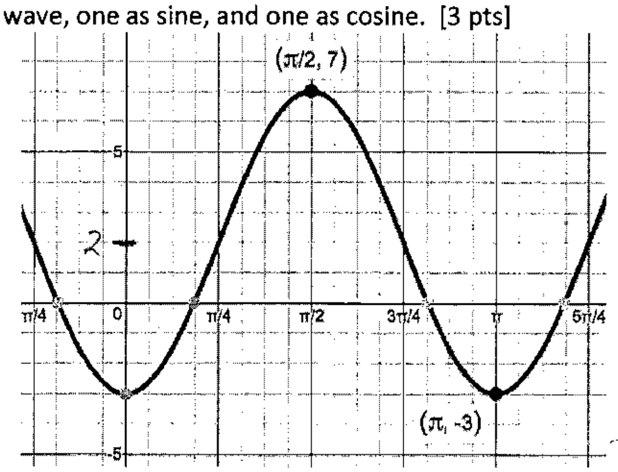
$\csc\theta = \frac{13}{-12}$

11. Graph once cycle of the function below.

$$y = -4\cos\left(\frac{1}{2}(x-\pi)\right) + 2$$



12. Given the graph, write two possible equations that match the wave, one as sine, and one as cosine. [3 pts]



$y = 5\sin(2(x - \pi/4)) + 2$

$y = -5\cos(2x) + 2$

13. Find an equation of a <u>negative</u> <u>cosine</u> function with amplitude=2, period=6π, passing through the point$\left(\frac{11\pi}{5}, -3\right)$.

$y = -2\cos\left(\frac{1}{3}\left(x - \frac{11\pi}{5}\right)\right) - 1$

Establish the identity.

14) $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$

$\cos\theta(\tan\theta + \cot\theta)$

$= \cos\theta\tan\theta + \cos\theta\cot\theta$

$= \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \sin\theta + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

Establish the identity.

15) $\csc\theta\tan\theta = \sec\theta$

$\csc\theta\tan\theta$

$= \left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

Solve each equation for θ on the interval $[0, 2\pi]$. Exact Answer.

16) $\csc^2\theta = -4\csc\theta - 4$

$\csc^2\theta + 4\csc\theta + 4 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$

$\csc\theta = -2$

$\sin\theta = -\frac{1}{2}$

$\theta = \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

17) $2\cos(3\theta) = -1$

$\cos(3\theta) = -\frac{1}{2}$

Let $x = 3\theta$

$\cos(x) = -\frac{1}{2}$

$x = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}\right\}$

Since $x = 3\theta$,

$\frac{1}{3}x = \theta$

Take this set and multiply by $\frac{1}{3}$ to get all θ's.

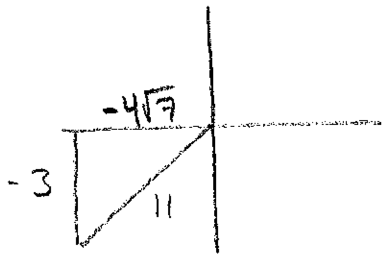
$\theta = \left\{\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}\right\}$

18) Draw a reference triangle given the following:

$\sin\theta = -\frac{3}{11} \quad \tan\theta > 0$

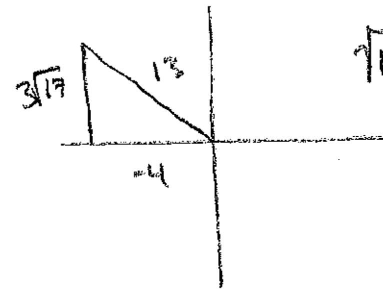
$\sqrt{112} = \sqrt{16\cdot 7}$

$= 4\sqrt{7}$



19) Draw a reference triangle given the following:

$cos^{-1}\left(-\frac{4}{13}\right)$



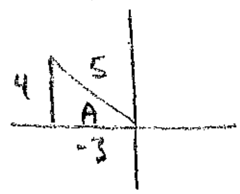
$\sqrt{153} = \sqrt{9\cdot 17}$

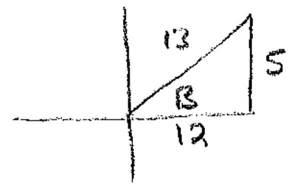
$= 3\sqrt{17}$

-- **END OF DAY 1 Material** --

-- **Start of DAY 2 Material** --

20) Find the **exact** value of the expression $tan\left(cos^{-1}\left(-\frac{3}{5}\right) + sin^{-1}\left(\frac{5}{13}\right)\right)$.

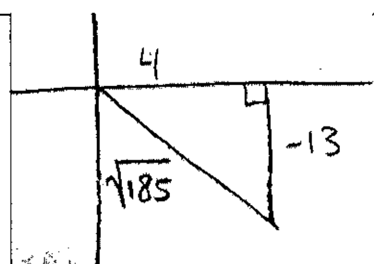




$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\left(\frac{4}{-3}\right) + \left(\frac{5}{12}\right)}{1 - \left(\frac{4}{-3}\right)\left(\frac{5}{12}\right)} = \frac{\frac{-16}{12} + \frac{5}{12}}{\frac{36}{36} - \frac{-20}{36}} = \frac{\frac{-11}{12}}{\frac{56}{36}}$$

$$= \frac{-11}{12} \cdot \frac{36}{56} = \frac{-396}{672} = \frac{-33}{56}$$

Use the reference triangle to the right on problems 8 and 9. Exact Answer.



21) $cos(2\theta)$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{4}{\sqrt{185}}\right)^2 - \left(\frac{-13}{\sqrt{185}}\right)^2$$

$$= \frac{16}{185} - \frac{169}{185} = \frac{-153}{185}$$

22) $sin(2\theta)$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{-13}{\sqrt{185}}\right)\left(\frac{4}{\sqrt{185}}\right)$$

$$= \frac{-104}{185}$$

23) Use a half-angle formula to find the **exact** value of the expression.

$$sin\left(\frac{7\pi}{8}\right)$$

Quad II (sin θ is positive)

$$\sin\left(\frac{7\pi}{8}\right) = \sin\left(\frac{1}{2}\cdot\frac{7\pi}{4}\right)$$

$$= \sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

24) Use a half-angle formula to find the **exact** value of the expression.

$$cot\left(\frac{17\pi}{8}\right) = \cot\left(\frac{\pi}{8}\right)$$

$$\cot\left(\frac{1}{2}\cdot\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{1-\cos(\pi/4)}$$

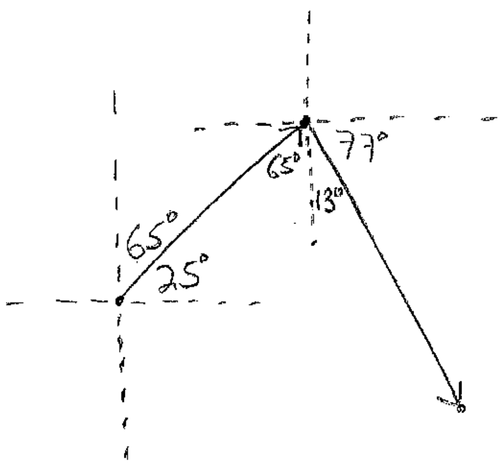
$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2}{2}-\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}$$

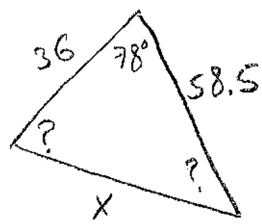
$$= \frac{\sqrt{2}}{2}\cdot\frac{2}{2-\sqrt{2}} = \frac{\sqrt{2}}{2-\sqrt{2}}$$

25) A sailboat leaves port on a heading of N65°E traveling at 12 knots for 3 hours. The boat then turns to a heading of S13°E traveling at 13 knots for 4.5 hours. **Approximate Answer.**

a) Draw a diagram of the boat's path.

b) Determine how far the boat is from where it left port.





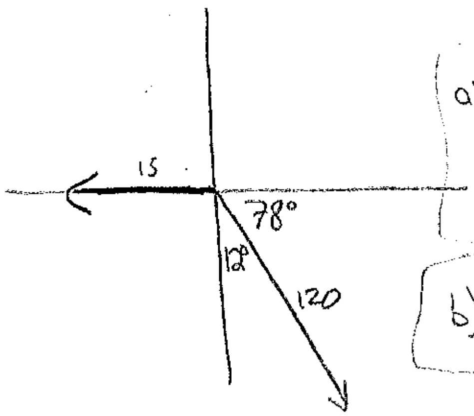
$$X^2 = 36^2 + 58.5^2 - 2(36)(58.5)\cos(78°)$$

$$X^2 = 3842.53$$

$$X = 61.99$$

It is 61.99 NM away from port

26) A small plane maintains a constant airspeed of 120 miles per hour (mph) headed S12°E. The wind is 15 mph in the west direction. **Approximate Answer.**
(a) Express the velocity $\boldsymbol{v_a}$ of the plane relative to the air and the velocity $\boldsymbol{v_w}$ of the wind in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.
(b) Find the velocity of the plane relative to the ground.
(c) Find the actual speed and direction of the plane relative to the ground.



a) $V_a = 120\cos(282)\,i + 120\sin(282)\,j$

$V_w = 15\cos(180)\,i + 15\sin(180)\,j$

b) $V_g = V_a + V_w = 9.95\,i - 117.38\,j$

c) Speed $= \|V_g\| = \sqrt{(9.95)^2 + (-117.38)^2}$

$= 117.8$ mph

Direction: $\tan^{-1}\left(\frac{-117.38}{9.95}\right) = -85.15°$

$$\begin{array}{r} 90 \\ -\ 85.15 \\ \hline 4.85 \end{array}$$

S 4.85° E

27a. The vector v has initial point P and terminal point Q. Write v in the form $ai + bj$; *that is, find its position vector.* P(-7,5) and Q (11, -12).

$-12 - 5 = -17 \leftarrow b$

$11 - (-7) = 18 \leftarrow a$

$\vec{v} = 18i - 17j$

27b. The vector v has initial point P and terminal point Q. Write v in the form $ai + bj$; *that is, find its position vector.* P(0,3) and Q (9, -14).

$9 - 0 = 9 \leftarrow a$

$-14 - 3 = -17 \leftarrow b$

$\vec{v} = 9i - 17j$

28) When referring to a vector, what does the term *magnitude* mean? How do you calculate it?

This is the length of the vector. You can calculate length/magnitude by using the pythagorean theorem.

A vector $\vec{w} = ai + bj$ has magnitude $= \sqrt{a^2 + b^2}$

29) Let $\vec{v} = 3i - 12j$ and $\vec{w} = -8i + 9j$

a) Calculate $4\vec{v}$

$4\vec{v} = 12i - 48j$

b) Calculate $3\vec{v} - 7\vec{w}$

$3\vec{v} = 9i - 36j$

$7w = -56i + 63j$

$3v - 7w = 65i - 99j$

c) Calculate $\|\vec{v} + \vec{w}\|$ **Approximate Answer.**

$v + w = -5i - 3j$

$\|v + w\| = \sqrt{34}$

$= 5.83$

30) Find the (a) magnitude and (b) direction angle of the vector $\langle 12, -18 \rangle$. **Approximate Answer.**

a) $\sqrt{12^2 + (-18)^2} = \sqrt{468} = 21.63$

b) $\tan^{-1}\left(\frac{-18}{12}\right) = -56.31$

$360 - 56.31$

$\theta = 303.69°$

31) Let $\vec{a} = -9i + 5j$ and $\vec{b} = -3i + 7j$. Find the angle between $\vec{a}$ *and* $\vec{b}$. **Approximate Answer.**

$\cos\theta = \frac{a \cdot b}{\|a\| \|b\|} = \frac{(-9)(-3) + (5)(7)}{(\sqrt{106})(\sqrt{58})} = \frac{27 + 35}{78.41} = \frac{62}{78.41}$

$\cos^{-1}\left(\frac{62}{78.41}\right) = 37.75°$

32) A student calculated the direction of the vector $v = 4i - 13j$. The answer they got was $-17.103°$.

a) What is wrong with presenting the solution this way? Correct the mistake.

They need to list the positive equivalent angle. Since it is in Quadrant IV, we need to do $360° - 17.103°$, which equals $342.897°$

b) When should the answer be presented as a direction using North/South/East/West?

Only when doing a word problem where directions are referred to as North/South/East/West.

So, $342.897°$ would then be S $72.897°$ E.