PC 7 – Review Problems

Show your work to earn full credit.

1a. The vector v has initial point P and terminal point Q. write v in the form $ai + bj$; *that is, find its position vector.* P(-7,5) and Q (11, -12). $11 - -7 = 18$ $-12 - 5 = -17$

$\vec{v} = 18i - 17j$

1b. The vector v has initial point P and terminal point Q. write v in the form $ai + bj$; *that is, find its position vector.* P(0,3) and Q (9, -14). $9 - 0 = 9$ $-14 - 3 = -17$

$\vec{v} = 9i - 17j$

1c. Explain, in words, what a position vector is AND how to find it given the initial and terminal point.

2. Let $\vec{v} = 3i - 12j$ and $\vec{w} = -8i + 9j$

a) Calculate $4\vec{v}$

$4(3) = 12$ $4(-12) = -48$

$4\vec{v} = 12i - 48j$

b) Calculate $3\vec{v} - 7\vec{w}$

$3\vec{v} = 9i - 36j$

$7\vec{w} = -56i + 63j$

$3\vec{v} - 7\vec{w} = 65i - 99j$

c) Calculate $\|\vec{v} + \vec{w}\|$

$\vec{v} + \vec{w} = -5i - 3j$

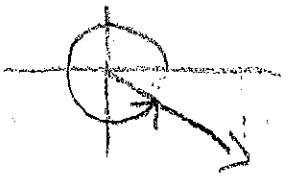
$\|\vec{v} + \vec{w}\| = \sqrt{34}$

$\|\vec{v} + \vec{w}\| \approx 5.83$

3. Find the (a) magnitude and (b) direction angle of the vector $\langle 12, -18 \rangle$.

Magnitude: $\sqrt{12^2 + (-18)^2} = \sqrt{468} = 21.63$

Direction Angle: $\theta = 303.69°$



$\tan^{-1}\left(\frac{-18}{12}\right) = -56.31°$

$360 - 56.31 = 303.69$

4. Let $\vec{a} = -9i + 5j$ and $\vec{b} = -3i + 7j$. Find the angle between $\vec{a}$ *and* $\vec{b}$.

$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{(-9)(-3) + (5)(7)}{(\sqrt{106})(\sqrt{58})} = \frac{62}{78.41} = 0.7907$

$\cos^{-1}(0.7907) = 37.75°$

5. Let $\vec{w} = 12i - 6j$ Find the unit vector $\vec{r}$ in the same direction as $\vec{w}$.

$\|w\| = \sqrt{180} = 6\sqrt{5}$

$\vec{r} = \frac{12}{6\sqrt{5}} i - \frac{6}{6\sqrt{5}} j$

$\vec{r} = \frac{2\sqrt{5}}{5} i - \frac{\sqrt{5}}{5} j$

6. Let $\vec{v} = -2i + 5j$ Find the vector $\vec{m}$ in the same direction as $\vec{v}$, but with magnitude 5.

$\|\vec{v}\| = \sqrt{29}$

$5\left(\frac{-2\sqrt{29}}{29} i + \frac{5\sqrt{29}}{29} j\right)$

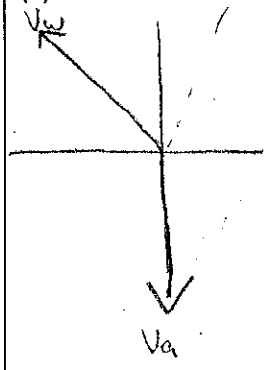
$\vec{m} = \frac{-10\sqrt{29}}{29} i + \frac{25\sqrt{29}}{29} j$

7. A small plane maintains a constant airspeed of 120 miles per hour (mph) headed due south. The wind is 15 mph in the northwestern direction.

(a) Express the velocity v_a of the plane relative to the air and the velocity v_w of the wind in terms of i and j.

(b) Find the velocity of the plane relative to the ground.

(c) Find the actual speed and direction of the plane relative to the ground.



a) $\vec{V}_a = 120\cos(270) i + 120\sin(270) j$

$\vec{V}_w = 15\cos(135) i + 15\sin(135) j$

b) $\vec{V}_g = -10.61 i - 109.39 j$

c) $\|\vec{V}_g\| = \sqrt{12078.74} = 109.91$ mph ← speed

$\tan^{-1}\left(\frac{-109.39}{-10.61}\right) = 84.46$

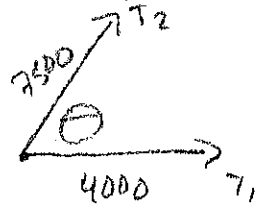
90 − 84.46 = 5.54°

S 5.54° W ← direction

8. A farmer wishes to remove a stump from a field by pulling it out with his tractor. Having removed many stumps before, he estimates that he will need 10,000 pounds of force to remove the stump. However, his tractor is only capable of pulling with a force of 4000 pounds, so he asks his neighbor to help. His neighbor's tractor can pull with a force of 7500 pounds. They attach the two tractors to the stump with a 65° angle between the tractors.

(a) Will they generate enough force to remove the stump? Explain.

(b) What if they used a 35° angle between the tractors, will they be successful in removing the stump then? Explain.



$T_1 = 4000\cos(0) i + 4000\sin(0) j$

$T_2 = 7500\cos(65) i + 7500\sin(65) j$

$T_1 + T_2 = 7169.64 i + 6797.31 j$

$\|T_1 + T_2\| = 9,879.63$ lbs

No! Not enough to remove the stump.

If $\theta = 35°$...

They'll be able to pull with 11,018.13 lbs!

and easily move the stump.