Name: Key Period: ____________

Ch. 5 Review **Show all work. Simplify answers.**

Establish the identity.

1) $tan^2\,\theta cos^2\,\theta + cot^2\,\theta sin^2\,\theta = 1$

$\tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta$

$= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta + \left(\frac{\cos^2\theta}{\sin^2\theta}\right)\sin^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Establish the identity.

2) $\frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta} = \frac{\cos\theta+1}{\sin\theta+1}$

$\frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta}$

$= \frac{(1+\sin\theta)}{(1+\sin\theta)} \cdot \frac{1-\sin\theta}{\cos\theta} + \frac{1}{1+\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)$

$= \frac{(1+\sin\theta)(1-\sin\theta) + \cos\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{1-\sin^2\theta + \cos\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{\cos^2\theta + \cos\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{\cancel{\cos\theta}(\cos\theta+1)}{(1+\sin\theta)(\cancel{\cos\theta})}$

$= \frac{\cos\theta+1}{1+\sin\theta}$

Solve each equation for θ on the interval $[0, 2\pi]$.

3) $\sqrt{\tan^2\theta} = \sqrt{\frac{1}{3}}$

$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}}$

$\tan\theta = \pm\frac{\sqrt{3}}{3}$

$\theta = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

4) $2\sin^2\theta - \sin\theta - 1 = 0$

$2x^2 - x - 1 = 0$

$-2x^2$, $-2x$, $1x$, $-x$

	x	-1
1	$1x$	-1
$2x$	$2x^2$	$-2x$

$(2x+1)(x-1) = 0$

$x = -\frac{1}{2}$ $\quad x = 1$

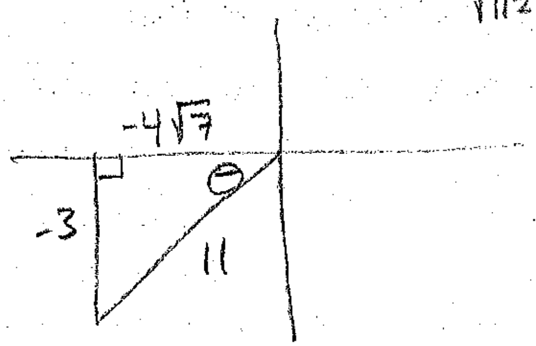
$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

5) Draw a reference triangle given the following:

$\sin\theta = -\frac{3}{11}$ $\quad \tan\theta > 0$

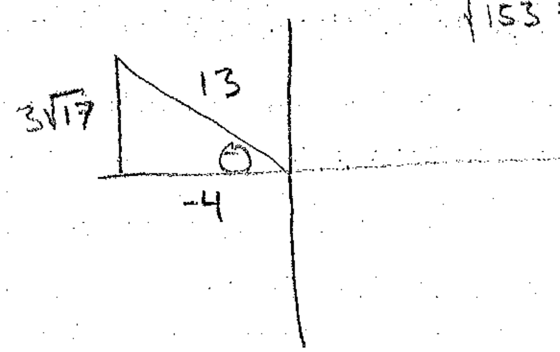
$\sqrt{112} = \sqrt{16 \cdot 7}$



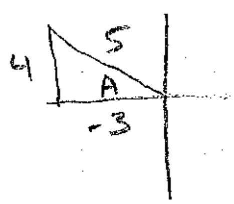
6) Draw a reference triangle given the following:

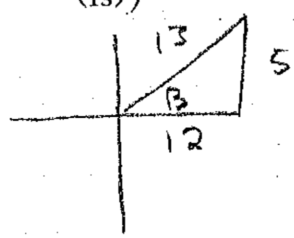
$\cos^{-1}\left(-\frac{4}{13}\right)$

$\sqrt{153} = \sqrt{9 \cdot 17}$



7) Find the exact value of the expression $\tan\left(\cos^{-1}\left(-\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$.



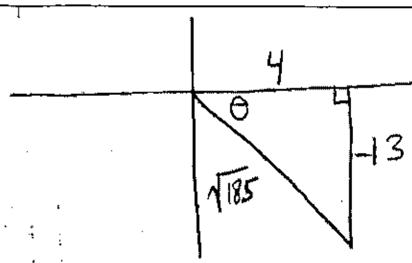


$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\left(\frac{4}{-3}\right) + \left(\frac{5}{12}\right)}{1 - \left(\frac{4}{-3}\right)\left(\frac{5}{12}\right)} = \frac{\left(\frac{4}{4}\right)\left(\frac{4}{-3}\right) + \frac{5}{12}}{1 - \frac{20}{-36}} = \frac{\frac{-16}{12} + \frac{5}{12}}{\frac{36}{36} + \frac{20}{36}} = \frac{\frac{-11}{12}}{\frac{56}{36}} = \frac{-11}{12} \cdot \frac{36}{56} = \frac{-396}{672}$

$= \frac{-33}{56}$

Use the reference triangle to the right on problems 8 and 9.



8) $cos(2\theta)$

$$= \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{4}{\sqrt{185}}\right)^2 - \left(\frac{13}{\sqrt{185}}\right)^2$$

$$= \frac{16}{185} - \frac{169}{185}$$

$$\boxed{= \frac{-153}{185}}$$

9) $cot(2\theta)$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{-13}{4}\right)}{1-\left(\frac{13}{4}\right)^2}$$

$$= \frac{\frac{-26}{4}}{\frac{16}{16}-\frac{169}{16}} = \frac{\frac{-26}{4}}{\frac{-153}{16}} = \frac{-26}{4}\cdot\frac{16}{-153}$$

$$= \frac{-416}{-612} = \frac{104}{153}$$

Flip it! $\frac{153}{104}$

10) Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{7\pi}{8}\right)$$

$\uparrow$ Quad II (sin is positive)

$$\sin\left(\frac{\frac{7\pi}{4}}{2}\right) = +\sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2}{2}}} \quad \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

11) Use a half-angle formula to find the exact value of the expression.

$$cot\left(\frac{17\pi}{8}\right)$$

$$\tan\left(\frac{\frac{17\pi}{4}}{2}\right) = \frac{1-\cos\frac{17\pi}{4}}{\sin\frac{17\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2}}$$

$$= \frac{2-\sqrt{2}}{\sqrt{2}}$$

$$\cot\left(\frac{17\pi}{8}\right) = \frac{\sqrt{2}}{2-\sqrt{2}}$$

Find the exact value of the expression $\tan[\cos^{-1}($[illegible]$)]$