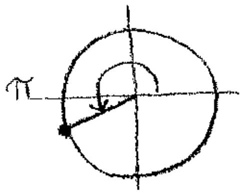
PC 5-5 Notes

Use a half-angle formula to find the exact value of the expression.

$$\sin\left(\frac{9\pi}{8}\right)$$

Step 1: What quadrant is $\frac{9\pi}{8}$ in?



Quadrant 3

Sine is negative in quadrant 3 so choose the negative formula.

$$\pm\sqrt{\frac{1-\cos A}{2}}$$

Step 2: Re-write as $\frac{A}{2}$.

$$\frac{9\pi}{8} = \frac{A}{2}$$

$$2\cdot\frac{9\pi}{8} = \frac{A}{\cancel{2}}\cdot\cancel{2}$$

$$\frac{9\pi}{4} = A$$

Step 3: Plug in values and simplify.

$$\sin\left(\frac{9\pi}{8}\right) = \sin\left(\frac{1}{2}\cdot\frac{9\pi}{4}\right) = -\sqrt{\frac{1-\cos\frac{9\pi}{4}}{2}}$$

$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{-\sqrt{2-\sqrt{2}}}{2}$$

Use a half-angle formula to find the exact value of the expression.

$$\tan\left(\frac{11\pi}{12}\right)$$

Use either formula!

I'll use $\tan\left(\frac{1}{2}A\right) = \frac{1-\cos A}{\sin A}$

Find A first

$$2\cdot\frac{11\pi}{12} = \frac{A}{2}\cdot 2$$

$$\frac{11\pi}{6} = A$$

$$\tan\left(\frac{11\pi}{12}\right) = \tan\left(\frac{1}{2}\cdot\frac{11\pi}{6}\right) = \frac{1-\cos\frac{11\pi}{6}}{\sin\frac{11\pi}{6}}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\frac{2-\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$= \frac{2-\sqrt{3}}{\cancel{2}}\cdot\frac{-\cancel{2}}{1} = -2+\sqrt{3}$$