Half-Angle Formulas

Directions:

Use a half-angle formula to find the exact value of the expression.

Half-Angle Formulas

$$sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - cos\alpha}{2}}$$

$$cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + cos\alpha}{2}}$$

$$tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos\alpha}{\sin\alpha}$$

Or...

$$tan\left(\frac{\alpha}{2}\right) = \frac{\sin\alpha}{1 + \cos\alpha}$$

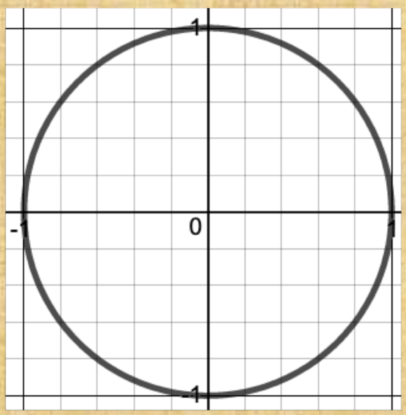
Half-Angle Formulas

$$sin\left(\frac{\alpha}{2}\right) = sin\left(\frac{1}{2}\alpha\right)$$

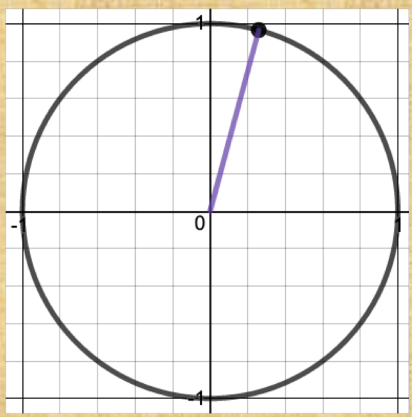
Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{5\pi}{12}\right)$$

Step 1: Find the appropriate half-angle formula and choose the $\pm$ based on where the given angle is

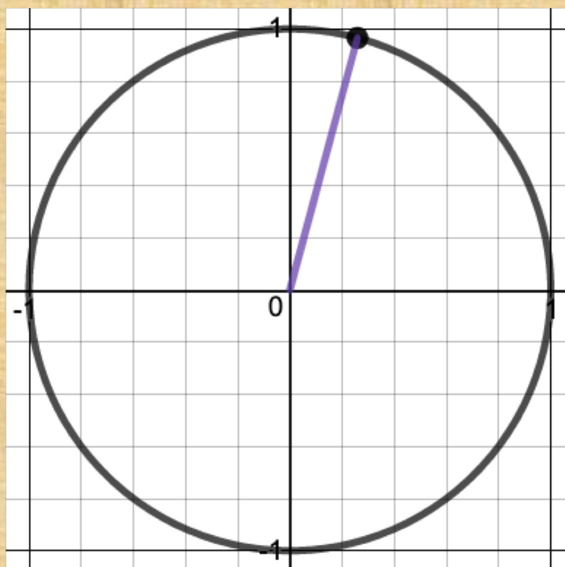


Where is $\frac{5\pi}{12}$?



Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{5\pi}{12}\right)$$



Since $\frac{5\pi}{12}$ is in quadrant 1, $sin\left(\frac{5\pi}{12}\right)$ will be positive. Therefore we use the positive (+) part of the sine half-angle formula and ignore the negative one.

$$sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - cos\alpha}{2}}$$

Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{5\pi}{12}\right)$$

$$\sin\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\cos\alpha}{2}}$$

Step 2: Re-Write your expression in the $\frac{\alpha}{2}$ form and figure out what α is.

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\alpha}{2}\right)$$

Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{5\pi}{12}\right)$$

$$\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\alpha}{2}\right)$$

$$\frac{5\pi}{12}=\frac{\alpha}{2}$$

$$2\cdot\frac{5\pi}{12}=\frac{\alpha}{2}\cdot 2$$

$$\frac{5\pi}{6}=\alpha$$

Use a half-angle formula to find the exact value of the expression.

$$sin\left(\frac{5\pi}{12}\right)$$

Step 3: Now that you have the correct formula and have found α, plug in the values and evaluate.

$$sin\left(\frac{5\pi}{12}\right) = sin\left(\frac{\alpha}{2}\right) = sin\left(\frac{\frac{5\pi}{6}}{2}\right) = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}-\left(\frac{-\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

So what about the other formulas for cosine and tangent?

Example: $\cos\left(\frac{11\pi}{12}\right)$

Complete the same steps as the previous example. Notice that in this problem, $\frac{11\pi}{12}$ is in quadrant 2. Since cosine is negative in that quadrant we would be using the negative formula for cosine.

$$-\sqrt{\frac{1+\cos\alpha}{2}}$$

Example: $\tan\left(\frac{11\pi}{12}\right)$

Tangent is slightly easier because you don't have to think about which quadrant the angle is in. Simply apply one of the two formulas and evaluate/simplify.

$$tan\left(\frac{\alpha}{2}\right)=\frac{1-\cos\alpha}{\sin\alpha}$$

$$tan\left(\frac{\alpha}{2}\right)=\frac{\sin\alpha}{1+\cos\alpha}$$

Note: Some problems can be solved using the sum and difference formulas or the half-angle formulas

Example: $\sin(15°)$

$15°$

could be re-written as either $45° - 30°$

or as

$\frac{30°}{2}$

When completing the many varieties of these trigonometry problems, be careful to read directions. It may be that you are given the choice to use any formula, but you might also be directed to use a specific one.