PC-1 5t Semester Final Review (**Extra Practice Problems**)

1) Algebraically find the domain of the following functions.

a) $h(x) = \frac{3x}{x^2-144}$

$x^2-144=0$

$\sqrt{x^2}=\sqrt{144}$

$x=12 \quad x=-12$

$D:\{x \mid x \neq 12, x \neq -12\}$

b) $f(x) = \frac{4}{x^2-19x}$

$x^2-19x=0$

$x(x-19)=0$

$x=0 \quad x=19$

$D:\{x \mid x \neq 0, x \neq 19\}$

c) $g(x) = \sqrt{21x+7}$

$21x+7 \geq 0$

$21x \geq -7$

$x \geq -\frac{7}{21}$

$x \geq -\frac{1}{3}$

$D:\{x \mid x \geq -\frac{1}{3}\}$

2) Find the domain of the function.

$m(x) = log_4(x+50)$

$x+50>0$

$x>-50$

$D:\{x \mid x>-50\}$

3) Find the domain of the function.

$n(x) = \log(12x-5)$

$12x-5>0$

$12x>5$

$x>5/12$

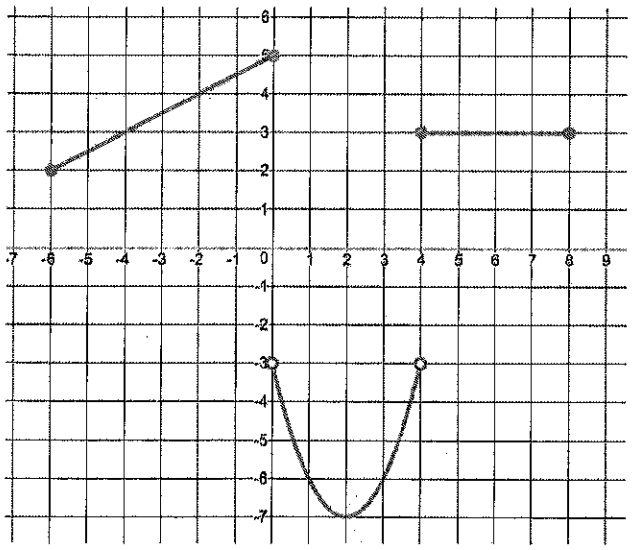
$D:\{x \mid x>\frac{5}{12}\}$

4) Find the domain of the function.

$y = 5^x$

All real numbers

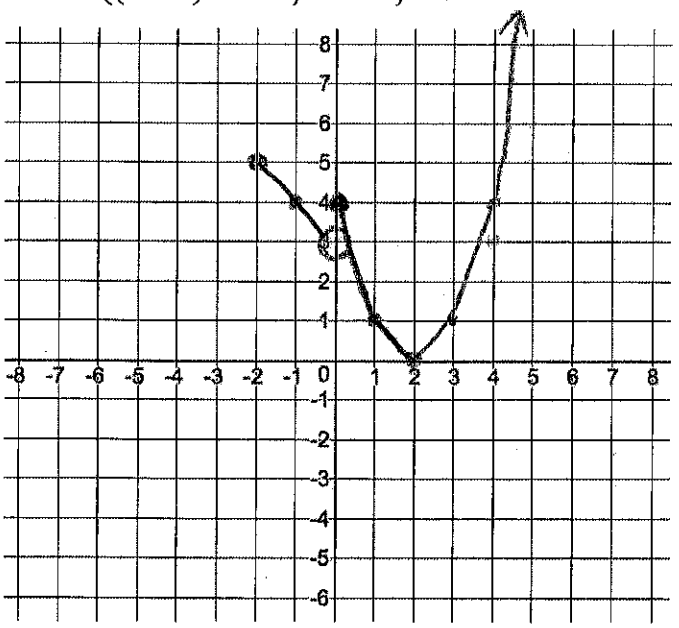
5) Write the definition for the piecewise function.



$$g(x)=\begin{cases} \frac{1}{2}x+5, & -6 \leq x \leq 0 \\ (x-2)^2-7, & 0<x<4 \\ 3, & 4 \leq x \leq 8 \end{cases}$$

6) ~~For the function find the:~~ Graph.

$$f(x)=\begin{cases} -x+3 & if \quad -2 \leq x < 0 \\ 4 & if \quad if \; x=0 \\ (x-2)^2 & if \quad if \; x>0 \end{cases}$$



7) Determine algebraically if the function is even, odd, or neither.

$$f(x) = x^5 + x^3$$

$f(-x) = (-x)^5 + (-x)^3 = -x^5 - x^3$ (NOT Even)

$-f(x) = -(x^5 + x^3)$

$= -x^5 - x^3$

ODD because $f(-x) = -f(x)$

8) Determine algebraically if the function is even, odd, or neither.

$$h(x) = \frac{2x}{x^2 + 5}$$

$h(-x) = \frac{2(-x)}{(-x)^2 + 5} = \frac{-2x}{x^2 + 5}$ (NOT Even)

$-h(x) = -\left(\frac{2x}{x^2+5}\right) = \frac{-2x}{x^2+5}$

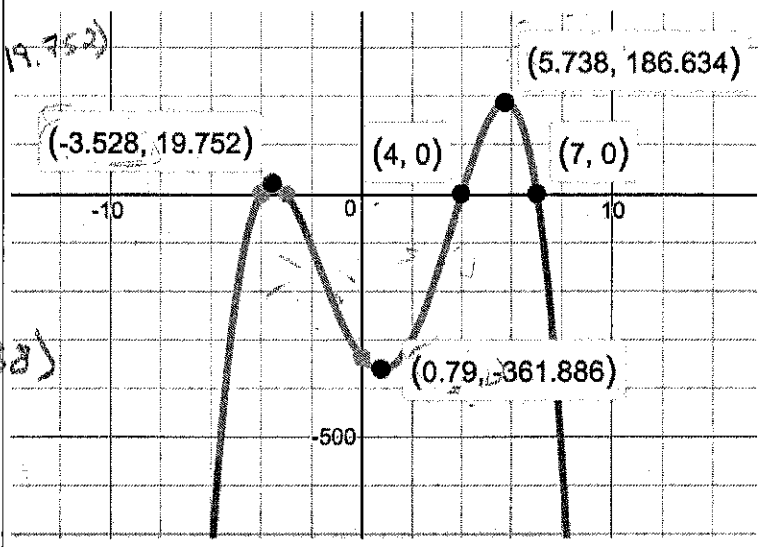
ODD because... $h(-x) = h(-x)$

Use the graph to the right to answer the following:

9) Identify the following (if present)

Local Maximum: (5.738, 186.634), (-3.528, 19.752)

Local Minimum: (0.79, -361.886)



10) Identify intervals for which the graph is increasing/decreasing.

Increasing: $(-\infty, -3.528) \cup (0.79, 5.738)$

Decreasing: $(-3.528, 0.79) \cup (5.738, \infty)$

11) Identify intervals for which the graph is concave up/down.

Concave Down: $(-\infty, -1.369) \cup (3.264, \infty)$

Concave Up: $(-1.369, 3.264)$

12) Find the Domain:

All Real #'s

13) Find the Range:

$R: \{y \mid y \leq 186.634\}$

14) Point $A(-9,25)$ is on the graph of $y = f(x)$. Determine the location of point A' after the transformation shown. Describe the transformation steps in detail (use words).

a) $\frac{1}{5}f(-3(x-7))$

$(-9,25) \rightarrow$ Divide y by 5 $(-9, 5)$

$(-9,5) \rightarrow$ Divide x by -3 $(3, 5)$

$(3,5) \rightarrow$ Add 7 to x $(10, 5)$

$(10, 5)$

b) $5f\left(\frac{1}{3}(x)\right) - 8$

$(-9,25) \rightarrow$ Multiply y by 5 $(-9, 125)$

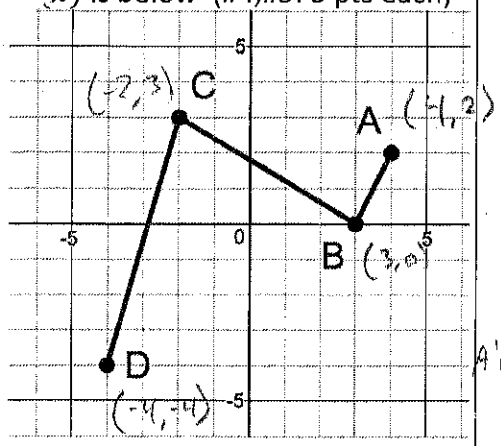
$(-9,125) \rightarrow$ Multiply x by 3 $(-27, 125)$

$(-27,125) \rightarrow$ Subtract 8 from y $(-27, 117)$

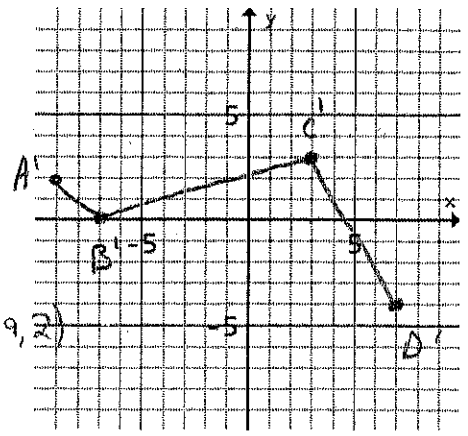
$(-27, 117)$

$(x, y) \to (-2x-1, y)$ $(x, y) \to (x+4, -2y-2)$

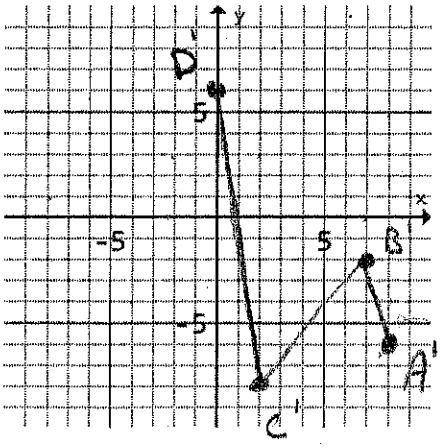
x) is below (#4,#5: 5 pts each)



15) Graph $h(-\frac{1}{2}(x+1))$



16) Graph $-2h(x-4)-2$



17) Let $n(x) = \frac{4x-7}{8x+2}$ Algebraically determine $n^{-1}(x)$.

$y = \frac{4x-7}{8x+2}$

$(8y+2)x = \frac{4y-7}{8y+2} \cdot (8y+2)$

$8yx + 2x = 4y - 7$

$-4y \quad -2x \quad -4y \quad -2x$

$8yx - 4y = -7 - 2x$

$8yx - 4y = -7 - 2x$

$\frac{y(8x-4)}{8x-4} = \frac{-7-2x}{8x-4}$

$y = \frac{-7-2x}{8x-4}$

18) Consider the polynomial function f with a root of $3i$. Find and list all roots.

$f(x) = x^4 + 4x^3 + 13x^2 + 36x + 36$

$(x-3i)(x+3i)$

	x	$3i$
$-3i$	$-3ix$	9
x	x^2	$3ix$

	x^2	$4x$	4
9	$9x^2$	$36x$	36
x^2	x^4	$4x^3$	$4x^2$

$(x^2+9)(x^2+4x+4)$

$4x^2$, $2x$, $2x$, $4x$

	x	2
2	$2x$	4
x	x^2	$2x$

$(x+2)(x+2)$

$X = \{3i, -3i, -2, -2\}$

You only need to list one -2.

19) Given a polynomial with the roots shown below, write a possible polynomial in standard form.

$x = \{3, 5 + 2i, 5 - 2i\}$

$(x-3)(x-5-2i)(x-5+2i)$

$-2i$			4
-5	$-5x$	25	
x	x^2	$-5x$	
	x	-5	$2i$

$(x-3)(x^2-10x+29)$

-3	$-3x^2$	$30x$	-87
x	x^3	$-10x^2$	$29x$
	x^2	$-10x$	29

$y = x^3 - 13x^2 + 59x - 87$

20) Given $R(x) = \frac{3(x-10)(x+2)}{x^2-2x-24}$, find the following. If none, write 'none'.

a) x-intercept(s)

$3(x-10)(x+2) = 0$

$(10, 0)$ $(-2, 0)$

b) y-intercept(s)

(Set $x = 0$)

$\frac{3(0-10)(0+2)}{0^2-2(0)-24} = \frac{3(-10)(2)}{-24}$

$\frac{-60}{-24} = \frac{10}{4} = \frac{5}{2}$

$(0, \frac{5}{2})$

c) vertical asymptote(s)

$x^2 - 2x - 24 = 0$

$24x^2$, $-6x$, $4x$, $-2x$

-6	$-6x$	-24
x	x^2	$4x$
	x	4

$(x-6)(x+4) = 0$

$x = 6$ $x = -4$

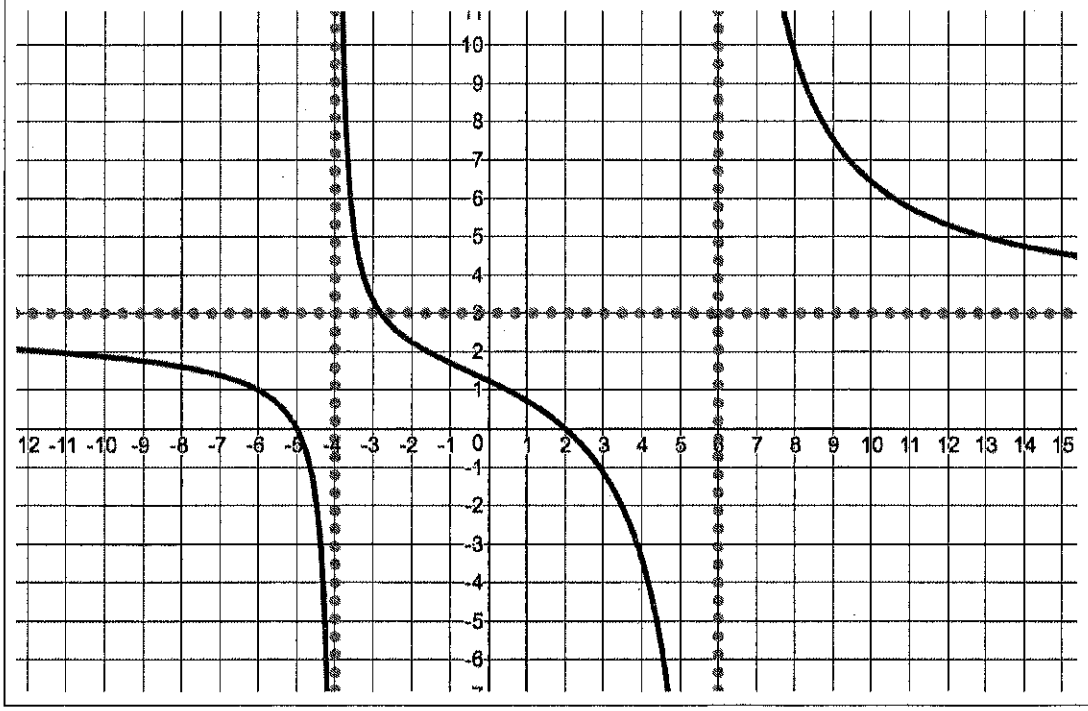
d) Horizontal asymptote

$y = 3$

e) Oblique Asymptote

None

21) Write a possible equation for the graph below. (5 pts)



$y = \frac{3(x+5)(x-2)}{(x+4)(x-6)}$

, Solve for x.

$5^{x+9} = 625^{4x}$

$5^{x+9} = (5^4)^{4x}$

$5^{x+9} = 5^{16x}$

$x+9 = 16x$

$-x \quad -x$

$\frac{9}{15} = \frac{15x}{15}$

$\frac{9}{15} = x$

$\frac{3}{5} = x$

23) Solve for x.

$\log(2x) + \log(x+1) = \log(12)$

$\log(2x(x+1)) = \log(12)$

$2x^2 + 2x = 12$

$\frac{2x^2 + 2x - 12 = 0}{2}$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ $\quad$ $x = 2$

extraneous

24) Solve for x.

$\log_7(x) + \log_7(3x-14) = 2$

$\log_7(3x^2 - 14x) = 2$

$7^2 = 3x^2 - 14x$

$0 = 3x^2 - 14x - 49$

$-147x^2$, $-21x$, $7x$, $-14x$

7	$7x$	-49
$3x$	$3x^2$	$-21x$
	x	-7

$(3x+7)(x-7) = 0$

$x = -\frac{7}{3}$ $\quad$ $x = 7$

extraneous

25) Alex deposits $14,322 into an account with a 2.6% interest rate compounded monthly. When will he have $22,000? Solve algebraically.

$\frac{22,000}{14322} = \frac{14,322\left(1 + \frac{.026}{12}\right)^{12t}}{14322}$

$1536 = (1.00217)^{12t}$

$\log_{1.00217} 1.536 = 12t$

$\frac{197.994}{12} = \frac{12t}{12}$

$16.4995 = t$

It ~ 16.5 years

26) An element, Jamiesonian – 33 , has a half-life of 3220 years. If there are 555 grams of this element to start, how long until only 230 grams remain?

$\frac{0.5A_0}{A_0} = \frac{A_0 e^{k(3220)}}{A_0}$

$0.5 = e^{k(3220)}$

$\ln(0.5) = k(3220)$

$-0.000215 = k$

$\frac{230}{555} = \frac{555e^{-0.000215t}}{555}$

$0.414 = e^{-0.000215t}$

$\frac{\ln(0.414)}{-0.000215} = \frac{-0.000215t}{-0.000215}$

$4101.811 = t$

In ~ 4101 years