Math 111 Final Exam Review

With the exception of rounding irrational logarithmic expressions and problems that specify that a calculator should be used, you should be prepared to do the entire problem without the use of a calculator.

1. Use the graph of $y = f(x)$ in Figure 1 to answer the following. Approximate where necessary.

(a) Evaluate $f(-1)$.

(b) Evaluate $f(0)$.

(c) Solve $f(x) = 0$.

(d) Solve $f(x) = -7$.

(e) Determine if f is even, odd, or neither from its graph.

(f) State any local maximums or local minimums.

(g) State the domain and range of f.

(h) Over what interval(s) is the function increasing?

(i) Over what interval(s) is the function decreasing?

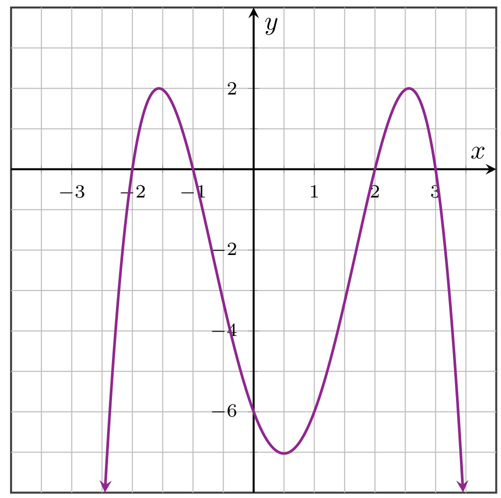
(j) Over what interval(s) is the function concave up?

(k) Over what interval(s) is the function concave down?

(l) Find the zeros of f.

(m) Find a possible formula for this polynomial function.

FIGURE 1



2. Let $f(x) = \dfrac{2x-6}{x+4}$.

(a) Find $f^{-1}(x)$.

(b) Confirm the inverse by computing $f^{-1}(f(x))$ and $f\left(f^{-1}(x)\right)$.

(c) State the domain and range of f and f^{-1}.

(d) Evaluate $f(0)$.

(e) Solve $f(x) = 3$.

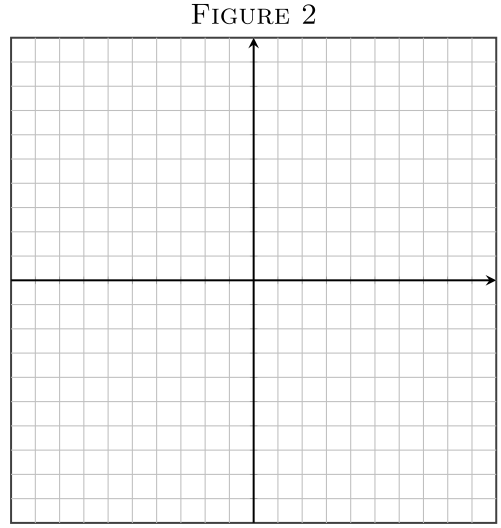
(f) Algebraically determine if f is even, odd, or neither.

(g) State any horizontal and vertical asymptotes.

(h) State any horizontal and vertical intercepts.

(i) Sketch a graph of $y = f(x)$ in Figure 2.

FIGURE 2



3. Let $f(x) = |x|$. For each of the following, sketch a graph of the transformation in Figure 4 and write the simplified formula for the function. Describe the order of transformations, being as specific as possible and listing them in an appropriate order.

(a) $y = -f(x)$
(b) $y = 2f(x)$
(c) $y = f(x) + 3$
(d) $y = 2f(x + 1) + 3$
(e) $y = f(3x)$
(f) $y = 3f(-2(x + 4)) - 1$

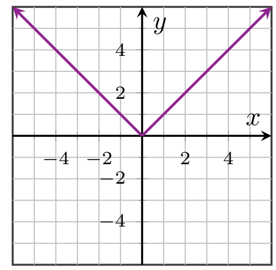


FIGURE 3. Graph of $y = |x|$

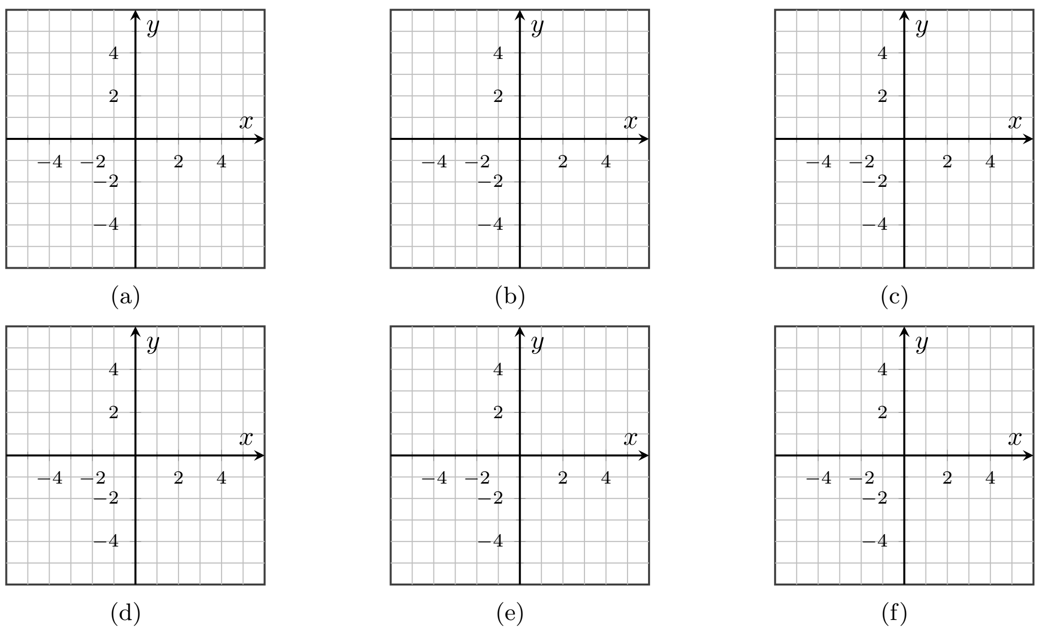


FIGURE 4

4. For each function below, identify the original (or basic) function and explain how the graph is a transformation of the graph of the original function. State all steps to this transformation in an appropriate order.

(a) $g(x) = -\frac{1}{3}\left(4(x - 7)\right)^3 - 2$
(b) $g(x) = 7\ln(x + 4) + 5$
(c) $g(x) = \sqrt{-\frac{1}{4}(x + 1)} - 3$

5. The point $(-4, 16)$ is on the graph of $y = f(x)$. Determine the point on the graph of...

(a) $y = f(x - 5) - 7$
(b) $y = -f(4x)$
(c) $y = 3f(-x)$
(d) $y = -\frac{1}{8}f(2(x + 3)) + 5$

6. Complete Table 1 below using the given values in the table. If any value is undefined, write "undefined."

TABLE 1

x	-4	-2	1	2	8
$f(x)$	8	0	-2	-4	-5
$g(x)$	3	4	5	6	3
$(g \cdot f)(x)$					
$(g \circ f)(x)$					
$f(x) + g(x)$					
$\frac{f(x)}{g(x)}$					
$f^{-1}(x)$					

7. Find a formula for the piecewise-defined function graphed in Figure 5 below.

$f(x) =$

8. In Figure 6, graph the piecewise function defined by

$$f(x) = \begin{cases} x^2 - 4, & -3 \le x < 0 \\ 2, & 0 < x < 1 \\ -\frac{1}{2}x + 2, & x \ge 1 \end{cases}$$

FIGURE 5. Graph of $y = f(x)$

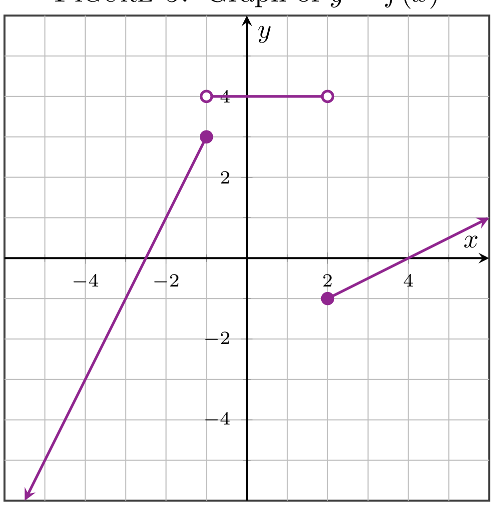
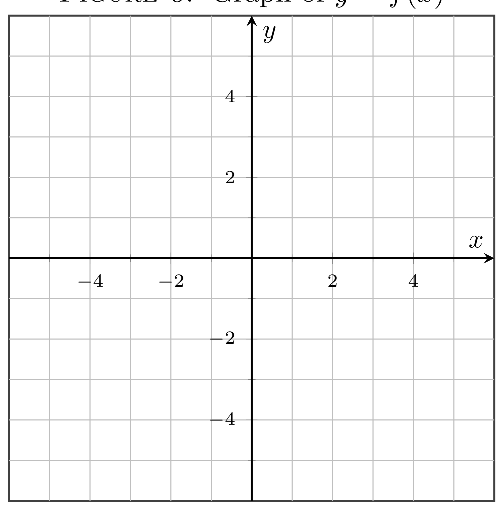


FIGURE 6. Graph of $y = f(x)$



9. The volume, $V(r)$ (in cubic centimeters) of a circular balloon of radius r (in centimeters) is given by $V(r) = \frac{4}{3}\pi r^3$. As someone blows air into the balloon, the radius of the balloon as a function of time t (in seconds) is given by $r = g(t) = 2t$.

(a) Find and interpret $V(3)$.

(b) Find and interpret $g(3)$.

(c) Find and interpret $V(g(3))$.

(d) Find and interpret $V(g(t))$.

(e) Explain why $g(V(r))$ in nonsense.

10. Find the following for the functions f, g, and h defined by

$$f(x) = \frac{2}{3x+1} \qquad g(x) = 3x^2 + 1 \qquad h(x) = 2x - 5$$

(a) $f(g(2))$

(b) $(h \circ f)(1)$

(c) $(h + g)(1)$

(d) $(g \circ g)(0)$

(e) $(f - g)(0)$

(f) $(g \cdot g)(x)$

(g) $(f \circ g)(x)$

(h) $(g \circ h)(x)$

(i) $(h \circ h)(x)$

11. Find the algebraic rule (or formula) of an exponential function that passes through each pair of points:

(a) $\left(-1, \frac{1}{3}\right)$ and $(1, 12)$

(b) $(2, 128)$ and $(5, 2)$

12. Find the exact values of the expressions below.

(a) $\log_4(64)$

(b) $\ln(\sqrt{e})$

(c) $\log_{10}\left(\frac{1}{100}\right)$

13. Solve the following equations. Give the exact solution and then round accurately to two decimal places. Clearly state each solution set.

(a) $7^x - 1 = 4$

(b) $e^{5x} = 10$

(c) $5e^x = 10$

(d) $3^{x^2} = 9^{x+4}$

(e) $3^{2x+1} = 6$

(f) $5 \cdot 7^x = 3 \cdot 2^{x-7}$

(g) $\log_4(2x+1) = 2$

(h) $\log_2(x) + \log_2(3) = \log_2(2)$

(i) $\log_2(x) - \log_2(3) = \log_2(2)$

(j) $2\log_5(x-6) = \log_5(x)$

(k) $\log_x\left(\sqrt{3}\right) = \frac{1}{4}$

(l) $\log(1-x) = 2 + \log(1+x)$

(m) $\log_6(x+4) + \log_6(x+3) = 1$

14. The percentage of carbon 14, Q, remaining in a fossil t years since decay began can be modeled by the function

$$Q = f(t) = 100e^{-0.000124t}$$

(a) If a piece of cloth is thought to be 750 years old. What percentage of carbon 14 is expected to remain in this sample?

(b) If a fossilized leaf contains 70% of its original carbon 14, how old is the fossil?

15. The temperature of a cup of tea after it was brewed can be modeled by the function $T(t) = 100e^{-0.1t} + 68$, where t is the number of minutes since the tea was brewed and $T(t)$ is the temperature in degrees Fahrenheit at time t.

(a) Find and interpret $T(0)$.

(b) Find and interpret $T(10)$.

(c) Solve and interpret $T(t) = 80$.

(d) Graph $y = T(t)$ in your calculator. What is the horizontal asymptote?

16. Tom and Jerry make separate investments at the same time. Their respective investments can be modeled by the functions

$$T(t) = 5000(1.065)^t \qquad \text{and} \qquad J(t) = 4500\left(1 + \frac{0.065}{12}\right)^{12t}$$

where t is the number of years since each investment began and $T(t)$ and $J(t)$ are their respective investment values in dollars.

(a) How much does Tom invest initially? How much does Jerry invest initially?

(b) What will the values of their respective investments be after 5 years?

(c) How long will it take for Jerry's investment to double?

17. The acidity of a solution is measured using the pH scale. Lucky for you, it's a logarithmic scale! The pH depends on the concentration, or molarity, in moles/liter of hydrogen ions. The formula for pH is given by $\text{pH} = -\log\left(\left[\text{H}^+\right]\right)$, where $[\text{H}^+]$ is the molarity of hydrogen ions. A low pH is very acidic (like lemons) and a high pH is very basic (like bleach).

(a) Coffee has a hydrogen ion concentration of about $\left[\text{H}^+\right] = 1.1 \times 10^{-5}$. Find the pH of coffee.

(b) Pure water is called neutral and has a pH of 7. Find the hydrogen ion concentration of pure water.

(c) The hydrogen ion concentration of bleach is about $\left[\text{H}^+\right] = 3.16 \times 10^{-13}$. Find the pH of bleach.

(d) The pH of pure Hydrochloric acid (HCl) is 0. Find the hydrogen ion concentration of HCl.

18. Let $g(x) = x^4 - 3x^3 + 4x$.

(a) Use your calculator to find the zeros of the function.

(b) Use your calculator to estimate the interval where the graph is concave down. Round accurately to the nearest tenth.

(c) Use your calculator to estimate the local maximum value and where it occurs.

(d) Use your calculator to estimate the absolute minimum value and where it occurs.

19. Find possible formulas for each polynomial function in Figure 7. Clearly list the zeros and their multiplicities.

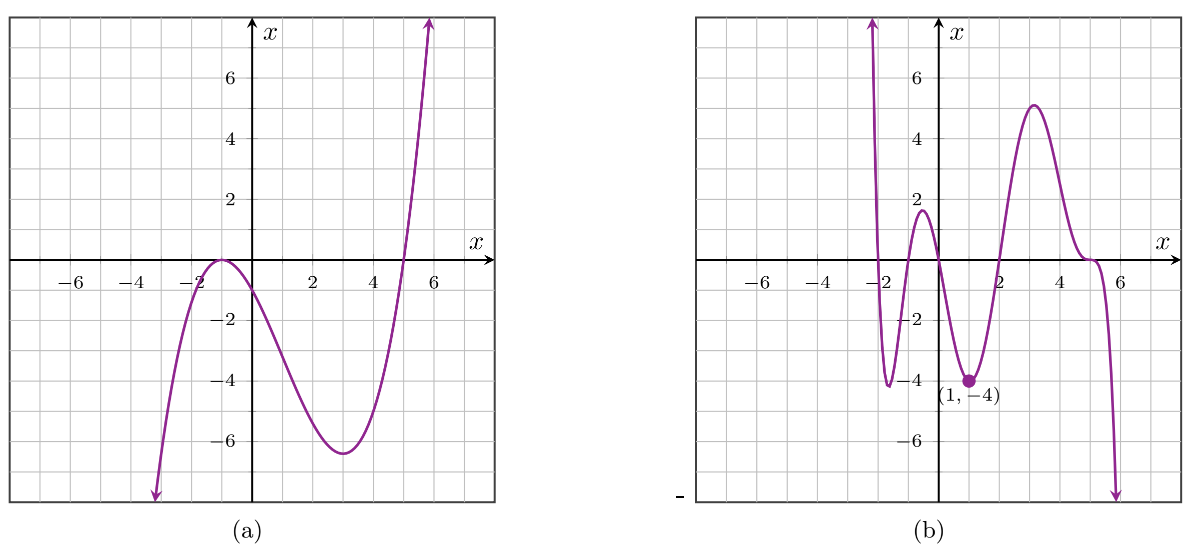


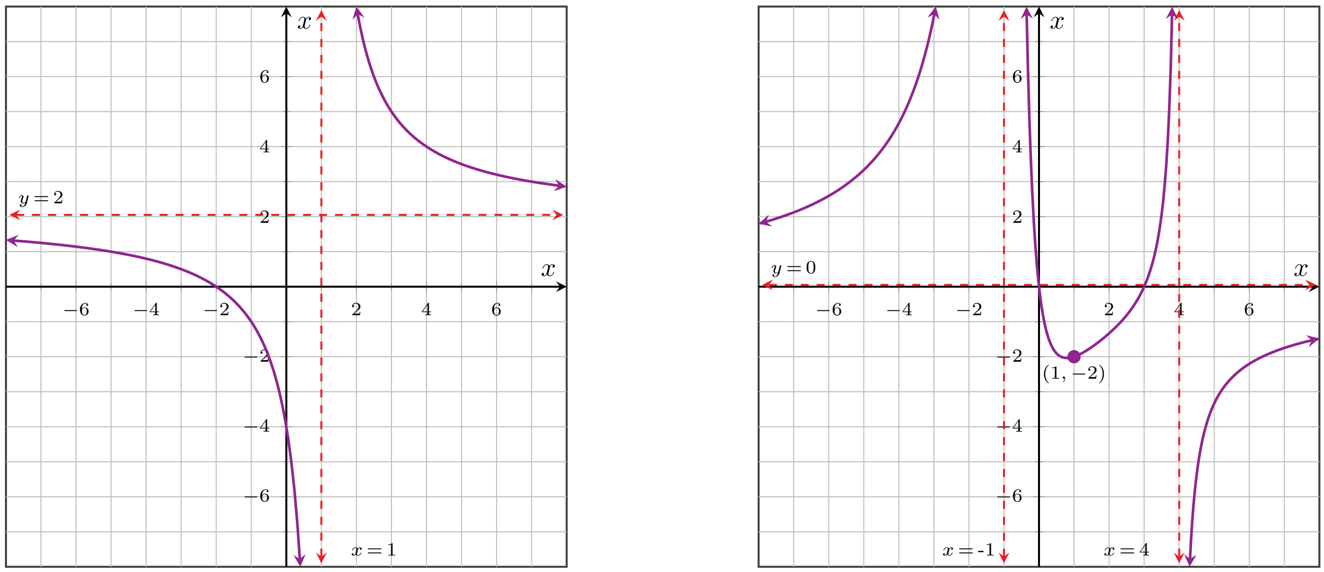
FIGURE 7

20. Sketch a graph of $y = f(x)$ for each polynomial function below. Also list the zeros and their multiplicities, the vertical intercept, and the long-run behavior.

(a) $f(x) = -x^2(x+4)$

(b) $g(x) = (x-2)(x+1)^2(x+2)$

21. Find possible formulas for each rational function in Figure 8 below. List the zeros and their multiplicities, any vertical asymptotes and any horizontal asymptotes.



(a) (b)

FIGURE 8

22. Find a possible formula for the rational function in Figure 9 below. List the zeros and their multiplicities, any vertical asymptotes and any horizontal asymptotes.

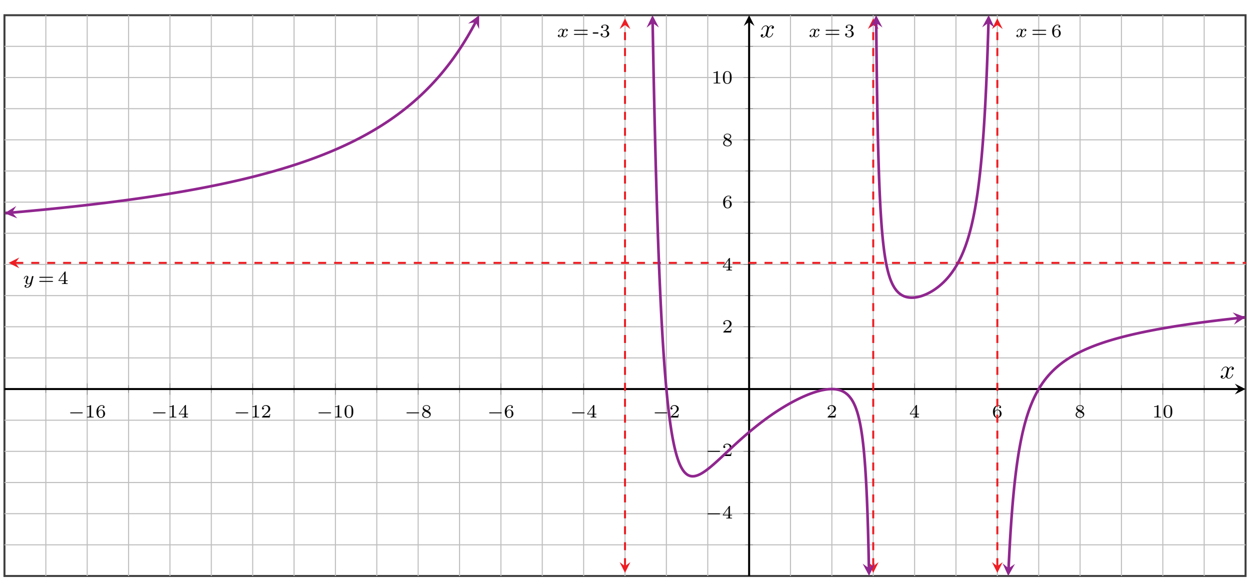


FIGURE 9