

PC 2-4 Notes

I can apply Descartes Rule of Signs to determine the number of positive, negative, and imaginary zeros of a polynomial. I can apply the Rational Zeros Theorem to a polynomial function to determine the potential Rational Zeros.

**Descartes Rule of Signs:**

Let  $f$  denote a polynomial function written in standard form.

Given a polynomial with integer coefficients written in standard form, the number of positive real zeros of  $f$  either equals the number of variations in the sign of the nonzero coefficients of  $f(x)$  or else equals that number less and even integer.

The number of negative real zeros of  $f$  either equals the number of variations in the sign of the nonzero coefficients of  $f(-x)$  or else equals that number less and even integer.

Example Application of Descartes Rule of Signs:

$$f(x) = x^3 - 4x^2 + 5x - 8$$

Given the polynomial below, use Descartes Rule of Signs to determine the number of positive, negative and imaginary roots.

1)  $f(x) = x^3 + 6x^2 + 11x + 6$

2)  $f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5$

3)  $f(x) = 2x^3 - 2x^2 + x + 5$

4)  $f(x) = 2x^3 + 11x^2 - 7x - 6$

5)  $f(x) = 7x^5 - 3x^3 + x - 20$

6)  $f(x) = x^5 + 3x^4 + x^3 - x^2 + x - 1$

**Rational Zeros Theorem**

Let  $f$  be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

$$\text{and } a_n \neq 0, a_0 \neq 0$$

where each coefficient is an integer. If  $\frac{p}{q}$ , in lowest terms is a rational zero of  $f$ , then  $p$  must be a factor of  $a_0$ , and  $q$  must be a factor of  $a_n$ .

Example application of the Rational Zeros Theorem:

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Given the polynomial below, create a list of all the potential rational zeros using the Rational Zeros Theorem.

1)  $f(x) = 3x^3 + 4x^2 - 5x - 2$

2)  $f(x) = 2x^3 + 3x - 5$

3)  $f(x) = 2x^3 + 11x^2 - 7x - 6$

4)  $f(x) = 3x^3 - 7x^2 - 14x + 24$

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