

PC 1-Review(2)

1) Find and fully simplify the difference quotient for the function g below. $\frac{g(x+h)-g(x)}{h}, h \neq 0$

$$g(x) = -3x + 4$$

$$\begin{aligned} \frac{g(x+h)-g(x)}{h} &= \frac{-3(x+h)+4 - (-3x+4)}{h} \\ &= \frac{-3x-3h+4+3x-4}{h} \\ &= \frac{-3h}{h} \\ &= -3 \end{aligned}$$

2) Find and fully simplify the difference quotient for the function f below. $\frac{f(x+h)-f(x)}{h}, h \neq 0$

$$f(x) = x^2 - 9x$$

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^2 - 9(x+h) - (x^2 - 9x)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 9x - 9h - x^2 + 9x}{h} \\ &= \frac{2xh + h^2 - 9h}{h} \\ &= h(2x + h - 9) \\ &= 2x + h - 9 \end{aligned}$$

3) Point $A(20, -4)$ is on the graph of $y = f(x)$. Determine the location of point A' after the transformation shown. Describe the transformation steps in detail.

a) $f((x+4)) - 3$

$(20, -4) \rightarrow$ Subtract 4 from $x \rightarrow (16, -4)$
 $(16, -4) \rightarrow$ Subtract 3 from $y \rightarrow (16, -7)$

$(20, -4)$ becomes $(16, -7)$ after the transformation

b) $-2f(4x)$

$(20, -4) \rightarrow$ Multiply y by $-2 \rightarrow (20, 8)$
 $(20, 8) \rightarrow$ Divide x by 4 $\rightarrow (5, 8)$

$(20, -4)$ becomes $(5, 8)$ after the transformation

4) Point $A(-9, 25)$ is on the graph of $y = f(x)$. Determine the location of point A' after the transformation shown. Describe the transformation steps in detail.

a) $\frac{1}{5}f(-3(x-7)) + 11$

$(-9, 25) \rightarrow$ Multiply y by $\frac{1}{5} \rightarrow (-9, 5)$
 $(-9, 5) \rightarrow$ Divide x by $-3 \rightarrow (3, 5)$
 $(3, 5) \rightarrow$ Add 7 to $x \rightarrow (10, 5)$
 $(10, 5) \rightarrow$ Add 11 to $y \rightarrow (10, 16)$

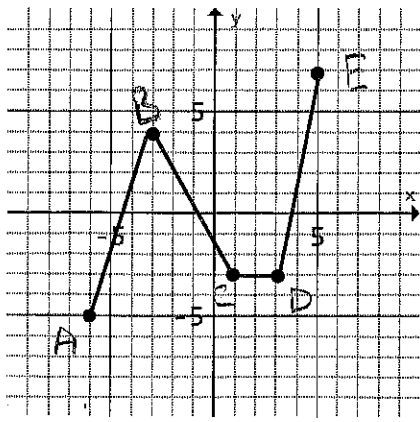
$(-9, 25)$ becomes $(10, 16)$ after the transformation

b) $5f\left(\frac{1}{3}(x+2)\right) - 8$

$(-9, 25) \rightarrow$ Multiply y by 5 $\rightarrow (-9, 125)$
 $(-9, 125) \rightarrow$ Multiply x by 3 $\rightarrow (-27, 125)$
 $(-27, 125) \rightarrow$ Subtract 2 from $x \rightarrow (-29, 125)$
 $(-29, 125) \rightarrow$ Subtract 8 from $y \rightarrow (-29, 117)$

$(-9, 25)$ becomes $(-29, 117)$ after the transformation

$h(x)$ is shown below

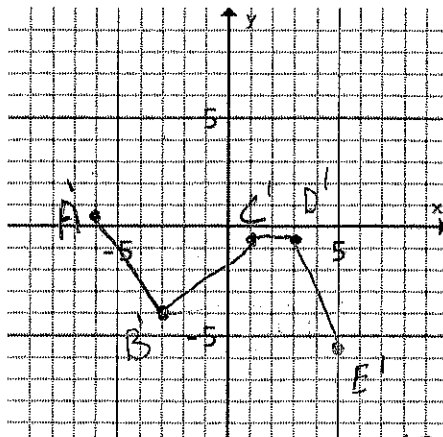


- A(-6, -5)
- B(-3, 4)
- C(1, -3)
- D(3, -3)
- E(5, 7)

5) a) Write the coordinate rule for the transformation: $-\frac{1}{2}h(x) - 2$

$$(x, y) \rightarrow (x, -\frac{1}{2}y - 2)$$

b) Graph it.

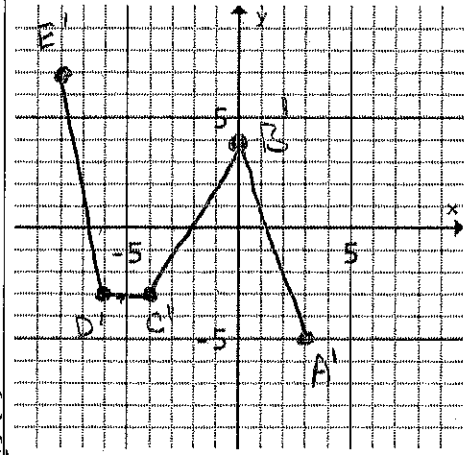


- A'(-6, 0.5)
- B'(-3, -4)
- C'(1, -0.5)
- D'(3, -0.5)
- E'(5, -5.5)

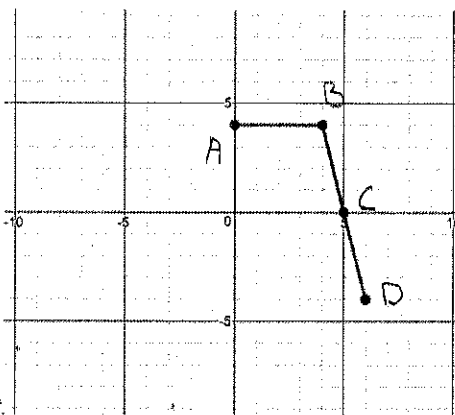
6) a) Write the coordinate rule for the transformation: $h(-x + 3)$

$$(x, y) \rightarrow (-x - 3, y)$$

b) Graph it.



$f(x)$ is shown below

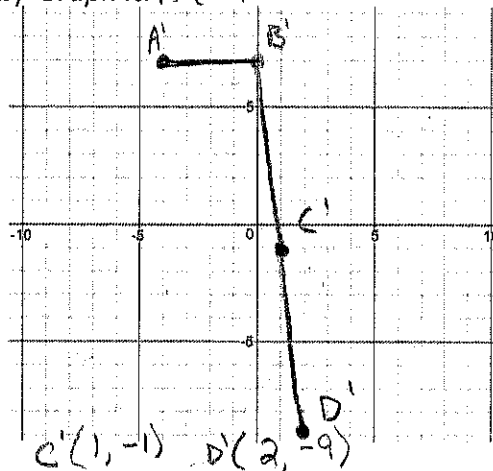


- A(0, 4)
- B(4, 4)
- C(5, 0)
- D(6, -4)

7) a) Write the coordinate rule for the transformation: $2f(x + 4) - 1$

$$(x, y) \rightarrow (x - 4, 2y - 1)$$

b) Graph it. A'(-4, 7) B'(0, 7)



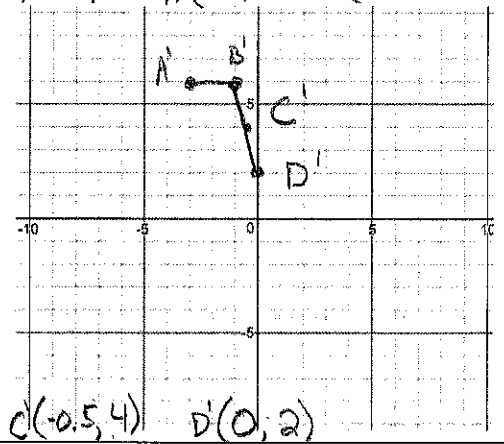
- C'(1, -1)
- D'(2, -9)

8) a) Write the coordinate rule for the transformation:

$$\frac{1}{2}f(2(x + 3)) + 4$$

$$(x, y) \rightarrow (\frac{1}{2}x - 3, \frac{1}{2}y + 4)$$

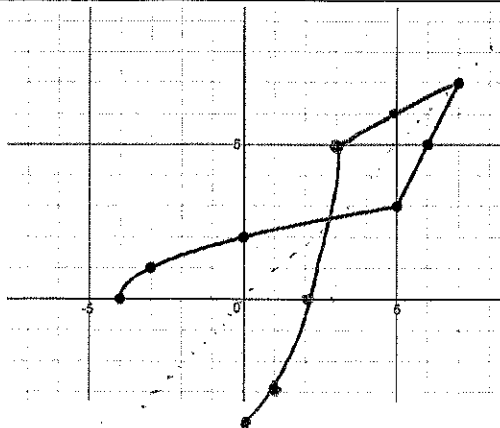
b) Graph it. A'(-3, 6) B'(-1, 6)



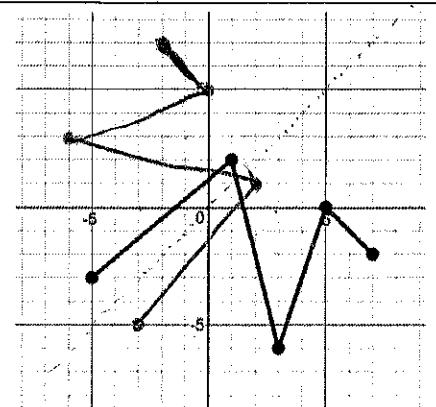
- C'(-0.5, 4)
- D'(0, 2)

Given the graph, graph its **inverse** on the same coordinate plane.

- (-4, 0)
- (-3, 1)
- (0, 2)
- (5, 3)
- (6, 5)
- (7, 7)
- (0, -4)
- (1, -3)
- (2, 0)
- (3, 5)
- (5, 6)



- (-5, -3)
- (1, 2)
- (3, -6)
- (5, 0)
- (7, -2)
- (-3, -5)
- (2, 1)
- (-6, 3)
- (0, 5)
- (-2, 7)



Let $n(x) = \frac{4x-7}{8x+2}$. Algebraically determine $n^{-1}(x)$.

$$y = \frac{4x-7}{8x+2}$$

$$x = \frac{4y-7}{8y+2}$$

$$(8y+2)x = \frac{4y-7}{8y+2} (8y+2)$$

$$8xy + 2x = 4y - 7$$

$$-4y \quad -2x \quad -4y \quad -2x$$

$$8xy - 4y = -7 - 2x$$

Domain of n :

$$D: \{x \mid x \neq -\frac{1}{4}\}$$

Range of n :

$$R: \{y \mid y \neq \frac{1}{2}\}$$

$$8xy - 4y = -7 - 2x$$

$$y(8x-4) = \frac{-7-2x}{8x-4}$$

$$y = \frac{-7-2x}{8x-4}$$

$$n^{-1}(x) = \frac{-7-2x}{8x-4}$$

Domain of $n^{-1}(x)$

$$D: \{x \mid x \neq \frac{1}{2}\}$$

Range of $n^{-1}(x)$

$$R: \{y \mid y \neq -\frac{1}{4}\}$$

7) Verify the inverse of $n(x) = \frac{4x-7}{8x+2}$ by showing that $n(n^{-1}(x)) = x$. Showing work is a major portion of this problem.

$$n(x) = \frac{4x-7}{8x+2}$$

$$n(n^{-1}(x)) = \frac{4(n^{-1}(x)) - 7}{8(n^{-1}(x)) + 2}$$

$$= \frac{4\left(\frac{-7-2x}{8x-4}\right) - 7}{8\left(\frac{-7-2x}{8x-4}\right) + 2}$$

$$= \frac{\frac{4(-7-2x)}{8x-4} - 7}{\frac{8(-7-2x)}{8x-4} + 2}$$

$$= \frac{\frac{-28-8x}{8x-4} - 7}{\frac{-56-16x}{8x-4} + 2}$$

$$= \frac{-28-8x}{8x-4} - \frac{7(8x-4)}{(8x-4)}$$

$$= \frac{-56-16x}{8x-4} + \frac{2(8x-4)}{8x-4}$$

$$= \frac{-28-8x}{8x-4} - \frac{56x-28}{8x-4}$$

$$= \frac{-56-16x}{8x-4} + \frac{16x-8}{8x-4}$$

$$= \frac{-28-8x-56x+28}{8x-4} = \frac{-64x}{8x-4}$$

$$= \frac{-56-16x+16x-8}{8x-4} = \frac{-64}{8x-4}$$

$$= \frac{-64x}{8x-4} \cdot \frac{8x-4}{-64} = \frac{-64x}{-64} = x$$

7) Let $f(x) = x + 4$ and $g(x) = 3x^2 + 7x - 20$, find each of the following. Simplify all answers.

a) $(f - g)(11)$

$$\begin{aligned} &= f(11) - g(11) \\ &= 11 + 4 - (3(11)^2 + 7(11) - 20) \\ &= 15 - (420) \\ &= -405 \end{aligned}$$

b) $(fg)(x)$

$$\begin{aligned} &= f(x) \cdot g(x) \\ &= (x + 4)(3x^2 + 7x - 20) \end{aligned}$$

4	$12x^2$	$28x$	-80
x	$3x^3$	$7x^2$	$-20x$
	$3x^2$	$7x$	-20

$$= 3x^3 + 19x^2 + 8x - 80$$

d) $(f \circ g)(-2)$

$$\begin{aligned} &= f(g(-2)) \\ &= f(x) = x + 4 \\ &= f(g(-2)) = (g(-2)) + 4 \\ &= (3(-2)^2 + 7(-2) - 20) + 4 \\ &= -18 \end{aligned}$$

e) $(g \circ f)(x)$

$$\begin{aligned} &= g(f(x)) \\ &= g(x) = 3x^2 + 7x - 20 \\ &= g(f(x)) = 3(f(x))^2 + 7(f(x)) - 20 \\ &= 3(x + 4)^2 + 7(x + 4) - 20 \\ &= 3x^2 + 31x + 56 \end{aligned}$$

f) $f + g$

$$\begin{aligned} &= x + 4 + 3x^2 + 7x - 20 \\ &= 3x^2 + 8x - 16 \end{aligned}$$

g) $\frac{f}{g}(3)$

$$\begin{aligned} &= \frac{f(3)}{g(3)} = \frac{3 + 4}{3(3)^2 + 7(3) - 20} = \frac{7}{28} \\ &= \frac{1}{4} \end{aligned}$$

8) Fill in the table below, then answer the questions using the table.

x	-5	-3	0	1	7
$f(x)$	10	2	-5	7	-3
$g(x)$	1	12	-3	-2	8
$f^{-1}(x)$	0	7	Undefined	Undefined	1

a) $f(-3) = 2$

d) $f^{-1}(-3) = 7$

g) $f^{-1}(f(-5)) = -5$

b) $g(f(7)) = 12$

e) $f(g(0)) = 2$

h) $g^{-1}(f(7)) = 0$

c) $g^{-1}(-3) = 0$

f) $g(f(-5)) = \text{Undefined}$

i) $f(f(0)) = 10$