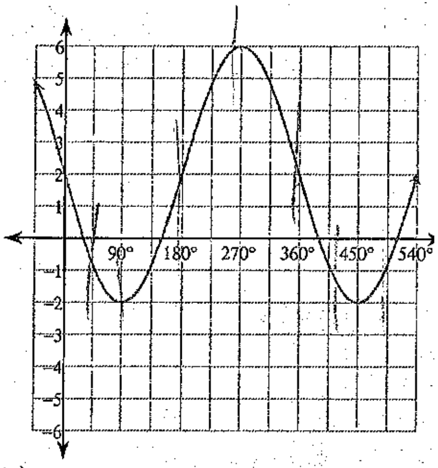
PC 1-9 Inverse Functions

1) Determine if $y = f(x)$ is a well-defined function. Why/Why not?

x	0	1	2	2	3
$f(x)$	-5	-4	3	7	7

No, $x=2$ has two different y-values.

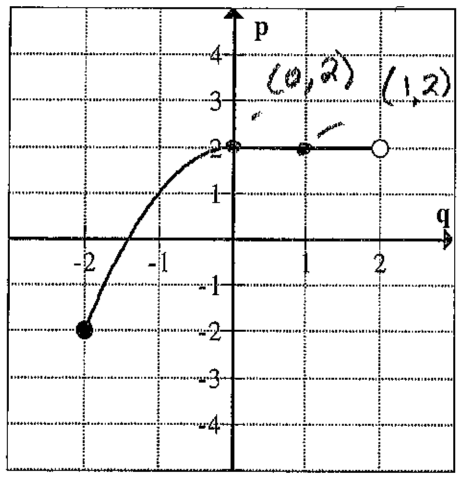
2) Determine if $y = g(x)$ is a well-defined function:



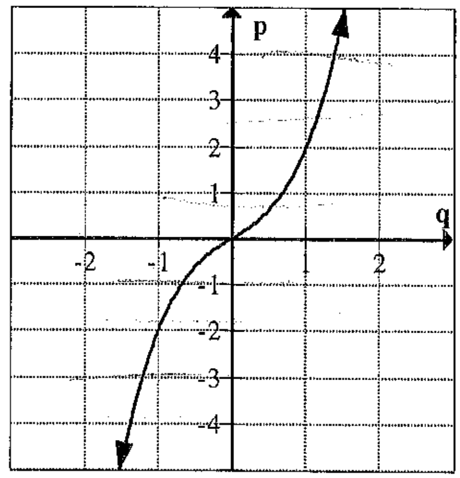
Yes, it passes the vertical line test.

A function, $p = f(q)$ is called **one-to-one** if every value of p in the range has a unique value of q in the domain associated to it. In other words, f is one-to-one if $f(x_1) = f(x_2)$ *automatically* implies that $x_1 = x_2$. One-to-one is often abbreviated 1-1, or written in fancy math lingo "injective". The **horizontal line test** states that if every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

3) Determine if the functions are 1-1.



No, there are some y-values with multiple x-values.

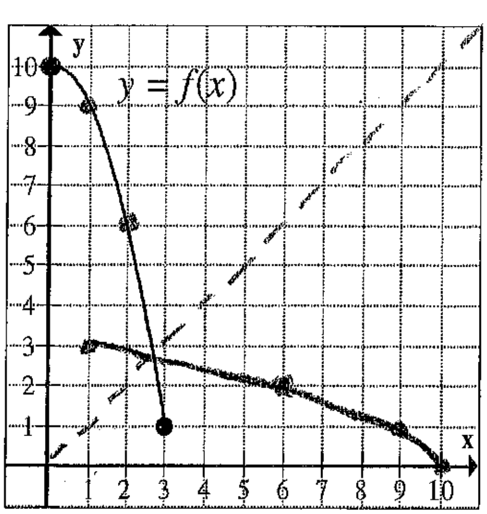


Yes, each y has only one x.

If y is a one-to-one function of x, then x will always be a one-to-one function of y. In this case, if $y = f(x)$, then the function $x = g(y)$ will be a new function where the roles of domain and range have reversed. The function g is called the **inverse function** of f.

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$

4) For the graph of the 1-1 function shown,



$f(x)$ - (0,10), (1,9), (2,6), (3,1)

$f^{-1}(x)$ - (10,0), (9,1), (6,2), (1,3)

a) Evaluate $f(1)$ = 9

b) Evaluate $f^{-1}(1)$ = 3

c) Evaluate $f(3)$ = 1

d) Evaluate $f(1)^{-1}$

$\frac{1}{f(1)} = \frac{1}{9}$

e) Solve $f(x) = 1$

$x = 3$

f) Evaluate $f(3^{-1})$

$f(\frac{1}{3}) \approx 9.7$

g) Evaluate $f^{-1}(6)$ = 2

h) Sketch $f^{-1}(x)$