

PC 1-9 Notes/Examples

Let $f(x) = \frac{2x+1}{x-1}$. Find the inverse.

$$y = \frac{2x+1}{x-1}$$

① Switch x and y

② Solve for y

$$x = \frac{2y+1}{y-1}$$

$$(y-1) \cdot x = \frac{2y+1}{y-1} \cdot (y-1)$$

$$xy - x = 2y + 1$$

$$-2y + x = -2y + 1 + x$$

$$xy - 2y = 1 + x$$

$$\frac{y(x-2)}{x-2} = \frac{1+x}{x-2}$$

$$y = \frac{1+x}{x-2}$$

$$\rightarrow f^{-1}(x) = \frac{1+x}{x-2}$$

Verify the inverse of $f(x) = \frac{2x+1}{x-1}$ by showing that

$$f(f^{-1}(x)) = x$$

$$f(x) = \frac{2x+1}{x-1}$$

$$f(f^{-1}(x)) = \frac{2(f^{-1}(x))+1}{(f^{-1}(x))-1}$$

$$= \frac{2\left(\frac{1+x}{x-2}\right) + 1}{\left(\frac{1+x}{x-2}\right) - 1} = \frac{2(1+x) + x-2}{\frac{1+x}{x-2} - \frac{x-2}{x-2}}$$

$$= \frac{2+2x+x-2}{x-2} = \frac{3x}{x-2}$$

$$\frac{1+x-x+2}{x-2} = \frac{3}{x-2}$$

$$\frac{3x}{x-2} \cdot \frac{3}{x-2} = \frac{3x}{x-2} \cdot \frac{(x-2)}{3} = \frac{3x}{3}$$

$$= x$$