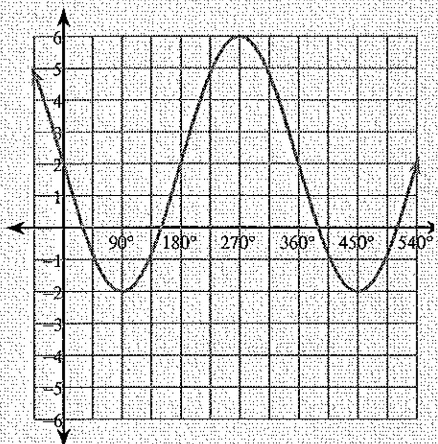
PC 1-9 Inverse Functions

1) Determine if $y = f(x)$ is a well-defined function. Why/Why not?

x	0	1	2	2	3
$f(x)$	-5	-4	3	7	7

No, x=2 has two outputs!

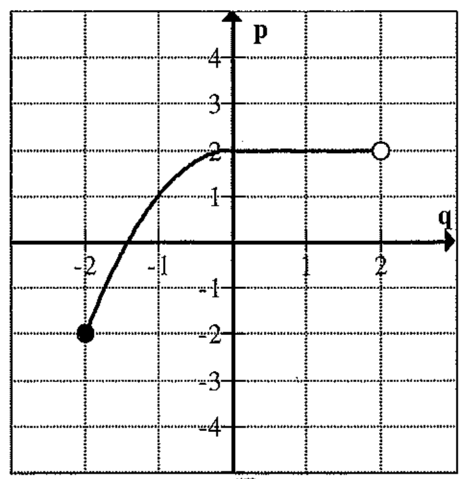
2) Determine if $y = g(x)$ is a well-defined function:



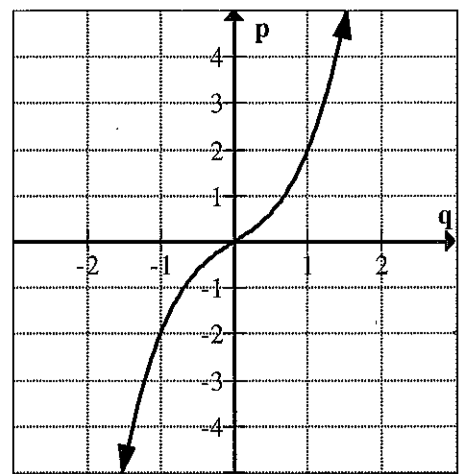
Yes, it passes the vertical line test.

A function, $p = f(q)$ is called **one-to-one** if every value of p in the range has a unique value of q in the domain associated to it. In other words, f is one-to-one if $f(x_1) = f(x_2)$ *automatically* implies that $x_1 = x_2$. One-to-one is often abbreviated 1-1, or written in fancy math lingo "injective". The **horizontal line test** states that if every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

3) Determine if the functions are 1-1.



Not 1-1. Fails the horizontal line test

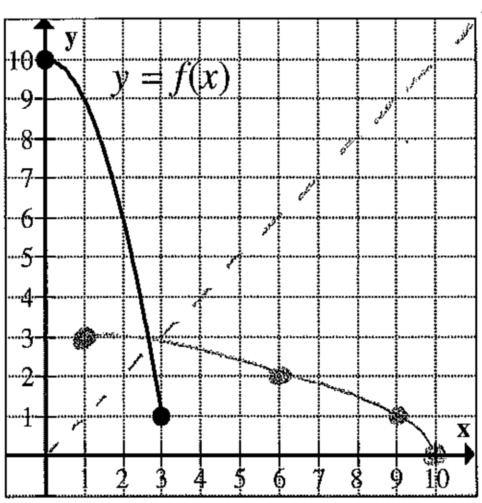


Yes 1-1. Passes the horizontal line test.

If y is a one-to-one function of x, then x will always be a one-to-one function of y. In this case, if $y = f(x)$, then the function $x = g(y)$ will be a new function where the roles of domain and range have reversed. The function g is called the **inverse function** of f.

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$ $\quad f(x)^{-1} = \frac{1}{f(x)}$

4) For the graph of the 1-1 function shown,



$(x, y) \to (y, x)$

a) Evaluate $f(1)$ = 9

b) Evaluate $f^{-1}(1)$ = 3 because

c) Evaluate $f(3)$ = 1

d) Evaluate $f(1)^{-1}$ = $\frac{1}{f(1)}$ = $\frac{1}{9}$

e) Solve $f(x) = 1$

$x = 3$

f) Evaluate $f(3^{-1})$ $f(\frac{1}{3})$ ≈ 9.7

g) Evaluate $f^{-1}(6)$ = 2

h) Sketch $f^{-1}(x)$

5) Let $f(q) = 2q + 1$. Find a formula for the inverse function of f using $f(f^{-1}(x)) = x$.

$y = 2x + 1$

switch x + y.

$x = 2y + 1$

Solve for y.

$\frac{x-1}{2} = \frac{2y}{2}$

$\frac{x-1}{2} = y$

$f^{-1}(q) = \frac{q-1}{2}$

Domain of f:	Domain of f^{-1}
All reals	All reals
Range of f:	**Range of f^{-1}**
All reals	All reals

6) Let $f(x) = \frac{x}{2x+1}$. Find a formula for the inverse function of f using $f(f^{-1}(x)) = x$.

$y = \frac{x}{2x+1}$

$x = \frac{y}{2y+1}$

$(2y+1)(x) = \frac{y}{(2y+1)} \cdot (2y+1)$

$2xy + x = y$
$\quad -x \qquad -y$

$2xy - y = -x$

$\frac{y(2x-1)}{(2x-1)} = \frac{-x}{(2x-1)}$ $\qquad y = \frac{-x}{2x-1}$

$f^{-1}(x) = \frac{-x}{2x-1}$

Domain of f:	Domain of f^{-1}
$D: \{x \mid x \neq -\frac{1}{2}\}$	$D: \{x \mid x \neq \frac{1}{2}\}$
Range of f:	**Range of f^{-1}**
$R: \{y \mid y \neq \frac{1}{2}\}$	$R: \{y \mid y \neq -\frac{1}{2}\}$

7) The function g is defined in the table shown. Fill in the missing entries.

x	-5	-2	-1	0	2	3	5
$g(x)$	3	-1	5	-5	0	-2	2
$g^{-1}(x)$							
$g^{-1}(g(x))$							

8) The total floor area of a green house (in ft^2) needed to grow p plants is $A = g(p)$. The cost, in \$, of constructing a greenhouse with floor area A is $C = f(A)$.

a) Interpret $g(400)$

b) Interpret $g^{-1}(400)$

c) Interpret $f(198) = 1090$

d) Interpret $f(g(1000)) = 1725$